# POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST) 

## Term-End Examination

June, 2016

## MSTE-002 : INDUSTRIAL STATISTICS ॥

Time: 3 hours
Maximum Marks : 50

## Note:

(i) Attempt all questions. Questions no. 2 to 5 have internal choices.
(ii) Use of scientific calculator is allowed.
(iii) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.
(iv) Symbols have their usual meaning.

1. State whether the following statements are true or false. Give reasons in support of your answers.
(a) If there are 2 equations having 3 variables in an LPP, then the maximum number of possible basic solutions is 3 .
(b) If the arrival rate is 6 per hour and service rate is 2 per hour, then the probability of no customer in queue is 0.7 .
(c) The estimated value of ' $b$ ' in the regression line $Y=a+b X+e$ in terms of variance-covariance is $\left[\operatorname{cov}(\mathrm{X}, \mathrm{Y}) / \sigma_{\mathrm{X}}^{2} \cdot \sigma_{\mathrm{Y}}^{2}\right]$.
(d) Ratio to trend method is used to estimate the trend values in a time series.
(e) Given the trend equation, $\hat{\mathrm{Y}}=108+2 \cdot 88 \mathrm{X}$ with 1980 as origin and yearly data from 1980 to 1982, the estimated value for 1985 is $119 \cdot 52$.
2. A manufacturer produces two types of models $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$. Each $\mathrm{M}_{1}$ model requires 4 hours of grinding and 2 hours of polishing; whereas each $\mathrm{M}_{2}$ model requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works for 60 hours a week. Profit on $\mathrm{M}_{1}$ model is ₹ 3 and on $\mathrm{M}_{2}$ model, it is ₹ 4 . Whatever is produced in a week is sold in the market.
(a) Formulate the problem mathematically to maximise the profit; and
(b) Using the graphical method, determine how the manufacturer should allocate his production capacity to the two types of models so that he may make the maximum profit in a week. $3+7$

OR

Use Big-M method to solve the following LPP :

## Maximise $4 \mathrm{x}_{1}+3 \mathrm{x}_{2}$

subject to the constraints :

$$
\begin{aligned}
2 x_{1}+x_{2} & \geq 10 \\
-3 x_{1}+2 x_{2} & \leq 6 \\
x_{1}+x_{2} & \geq 6 \\
x_{1} \geq 0 \text { and } x_{2} & \geq 0
\end{aligned}
$$

3. A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulties. His estimate of time for each man to perform each task, is given in the table below :

## Men

| Tasks | E | F | G | H |
| :---: | :---: | :---: | :---: | :---: |
| A | 18 | 26 | 17 | 11 |
| B | 13 | 28 | 14 | 26 |
| C | 38 | 19 | 18 | 15 |
| D | 19 | 26 | 24 | 10 |

How should the tasks be allocated to each subordinate so as to minimise the total man-hours?

## OR

(a) A book binder has one printing press, one binding machine and the manuscripts of a number of different books. The time required to perform the printing and binding operations for each book are given below :

| Book | Printing time <br> (in hours) | Binding time <br> (in hours) |
| :---: | :---: | :---: |
| 1 | 30 | 80 |
| 2 | 120 | 100 |
| 3 | 50 | 90 |
| 4 | 20 | 60 |
| 5 | 90 | 30 |
| 6 | 110 | 10 |

Determine the order in which the books should be produced, in order to minimise the total time.
(b) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, calculate (i) the mean queue size; and (ii) the probability that the queue size exceeds 10. If the input of the train increases to an average of 33 per day, what will the change in (i) and (ii) be ?
4. A study was conducted on the effect of temperature ( X ) on the yield of a chemical process (Y). The following data (in coded form) were collected :

| $X$ | $Y$ |
| :---: | :---: |
| -5 | -1 |
| -4 | 5 |
| -3 | 4 |
| -2 | 7 |
| -1 | 10 |
| 0 | 8 |
| 1 | 9 |
| 2 | 13 |
| 3 | 14 |
| 4 | 13 |
| 5 | 18 |

(a) Assuming a model $\mathrm{Y}=\mathrm{a}+\mathrm{bX}+\mathrm{e}$, what are the least square estimates of $a$ and $b$ ?
(b) Calculate the variances of the estimated regression coefficients.
(c) Test the hypothesis that the temperature (X) has no effect on the yield ( Y ) of the chemical process, i.e., $\mathrm{H}_{0}: \mathrm{b}=0$; against $\mathrm{H}_{1}: \mathrm{b} \neq 0$ at $\alpha=0.05$.

A statistician collected data of 78 values with two independent variables $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$. The four models considered are
(a) $\mathrm{Y}=\mathrm{B}_{0}+\mathrm{e}$,
(b) $\mathrm{Y}=\mathrm{B}_{0}+\mathrm{B}_{1} \mathrm{X}_{1}+\mathrm{e}$,
(c) $Y=B_{0}+B_{1} X_{1}+B_{2} X_{2}+e$, and
(d) $\mathrm{Y}=\mathrm{B}_{0}+\mathrm{B}_{2} \mathrm{X}_{2}+\mathrm{e}$.

The results are $\mathrm{SS}\left(\mathrm{B}_{0}\right)=652 \cdot 42, \mathrm{SS}\left(\mathrm{B}_{0}, \mathrm{~B}_{1}\right)=679 \cdot 34$, $\mathrm{SS}\left(\mathrm{B}_{0}, \mathrm{~B}_{2}\right)=654 \cdot 00, \mathrm{SS}\left(\mathrm{B}_{0}, \mathrm{~B}_{1}, \mathrm{~B}_{2}\right)=687.79$ and $\hat{\sigma}^{2}=0.91$. Find the additional contribution of
(i) $\mathrm{X}_{2}$ over $\mathrm{X}_{1}$, and (ii) $\mathrm{X}_{1}$ over $\mathrm{X}_{2}$. Test whether their inclusion in the model is justified.
5. Determine the seasonal indices for the data given below for the average quarterly prices of a commodity for four years :

| Years | Quarter I | Quarter II | Quarter III | Quarter IV |
| :---: | :---: | :---: | :---: | :---: |
| 2009 | 554 | 590 | 616 | 653 |
| 2010 | 472 | 501 | 521 | 552 |
| 2011 | 501 | 531 | 553 | 595 |
| 2012 | 403 | 448 | 460 | 480 |

## OR

(a) Fifteen successive observations on a stationary time series are as follows :
$34,24,23,31,38,34,35,31,29,28,25$, $27,32,33,30$

Calculate $\mathrm{r}_{1}, \mathrm{r}_{2}, \ldots, \mathrm{r}_{5}$ and plot the correlogram.
(b) For the model

$$
(1-0.2 B)(1-B) X_{t}=(1-0.5 B) a_{t},
$$

find $p, d, q$, and express it as ARIMA (p, d, q). Determine whether the process is stationary.

