# POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST) 

## ロロ65s

Term-End Examination<br>June, 2016

## MST-004 : STATISTICAL INFERENCE

Time : 3 hours
Maximum Marks : 50
Note:
(i) Attempt all questions. Questions no. 2 to 5 have internal choices.
(ii) Use of scientific calculator is allowed.
(iii) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.
(iv) Symbols have their usual meaning.

1. State whether the following statements are True or False. Give reasons in support of your answer. $5 \times 2=10$
(a) If degrees of freedom of a $t$-distribution is 5 , then its mean is 5 .
(b) An estimator ( T ) is said to be consistent, if its $\operatorname{var}(T)=\theta$.
(c) Rejecting the null hypothesis $\left(\mathrm{H}_{0}\right)$ when it is not true is called type-I error.
(d) The number of runs in the sequence PQQPPPQPQPQPP is 9 .
(e) If estimators $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$ of a parameter $\theta$ have variances $\frac{1}{n}$ and $\frac{5}{n}$, respectively, then $T_{1}$ is more efficient than $T_{2}$.
2. (a) Differentiate between the following with examples:
(i) Estimator and Estimate
(ii) Parameter and Statistic
(b) A manufacturer of mobiles has determined from experience that $3 \%$ of the mobiles he produces are defective. If a random sample of 100 mobiles is examined and sample proportion is calculated, then find
(i) the mean and standard error of the sampling distribution of the proportion defective.
(ii) the probability that the proportion defective is between 0.02 and 0.04 .

OR
(a) Write any two applications of the chi-square distribution.

The probability density function (pdf) of the chi-square distribution is given by

$$
\mathrm{f}\left(\chi^{2}\right)=\frac{1}{96} \mathrm{e}^{-\chi^{2 / 2}\left(\chi^{2}\right)^{3} ; 0<\chi^{2}<\infty}
$$

(i) What is the degree of freedom of this chi-square distribution?
(ii) What are the mean and variance of the chi-square distribution?
(b) Consider a population ${ }^{\bullet}$ consisting of 4 villages, the areas (in hectares) of which are given below :

| Village | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| Area | 805 | 425 | 610 | 587 |

(i) How many samples of size 3 are possible without replacement ? Write them.
(ii) How many samples of size 3 are possible with replacement?
3. (a) Show that the sample mean $(\overline{\mathrm{X}})$ is an unbiased estimator of the population mean $(\mu)$. To determine the average amount of money spent by the students of a university on textbooks, a random sample of 10 students was chosen and the students were questioned. If the amounts spent were $1000,1520,5600,7110,2270,1460,4220$, $6800,520,2120$,
what is the point estimate of the average amount spent by all students of the university?
(b) A company produces electric bulbs. To estimate the average life of the electric bulbs, the quality inspector of the company selects 80 bulbs randomly. She finds that the average life of these bulbs is 2500 hours and the standard deviation is 50 hours. Find $95 \%$ confidence limits for population average life of the electric bulbs produced by the company.

## OR

(a) Derive the maximum likelihood estimator for the parameter $\lambda$ of the Poisson distribution.
(b) If the number of weekly accidents occurring on a one kilometer stretch of a particular road follows the Poisson distribution with parameter $\lambda$, find the maximum likelihood estimate of $\lambda$ on the basis of the following data :

| Number of <br> Accidents | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 5 | 10 | 10 | 4 | 2 | 1 |

4. (a) A company uses two sources of raw materials. Both sources seem to have similar characteristics but the company is not sure about their respective uniformity. To check it, a sample of 10 lots from source A and another sample of 11 lots from source $B$ are taken. The following data are obtained :

|  | Source A | Source B |
| :--- | :---: | :---: |
| Sample mean | 94 | 88 |
| Sum of Squares <br> from their mean | 126 | 100 |

Is it likely that the variance of source $A$ is significantly greater than the variance of source $B$ at $\alpha=0.01$ ?
(b) The proportions of literates between groups of people of two districts, A and B, are tested. Out of 100 persons selected at random from each district, 55 from district A and 45 from district B are found to be literates. Test whether the proportion of literate persons in the two districts, A and B, are the same at $5 \%$ level of significance.

## OR

A company is trying to improve the work efficiency of its employees. It has organised a special training programme for all employees. In order to assess the effectiveness of the training programme, the company has selected 10 employees randomly and administered a well-structured questionnaire. The scores (out of 50) obtained by the employees are given in the following table :

| Employee <br> Number | Before <br> Training | After <br> Training |
| :---: | :---: | :---: |
| 1 | 30 | 35 |
| 2 | 32 | 34 |
| 3 | 37 | 32 |
| 4 | 34 | 33 |
| 5 | 36 | 33 |
| 6 | 33 | 37 |
| 7 | 39 | 37 |
| 8 | 33 | 42 |
| 9 | 30 | 40 |
| 10 | 32 | 43 |

It is known that the scores of the employees before and after the training programme follow the normal distributions. To examine whether the training programme has improved the efficiency of employees, answer the following :
(i) Are both samples paired or independent? 1
(ii) State the null and alternative hypotheses. 1
(iii) Which test is used for testing the null hypothesis in this case and why?
(iv) Conduct the suitable test at $1 \%$ level of significance and interpret the result.
5. The following contingency table presents the analysis of 300 persons according to hair colour and eye colour :

| Hair Colour | Eye Colour |  |  |
| :---: | :---: | :---: | :---: |
|  | Blue | Grey | Brown |
| Fair | 30 | 10 | 40 |
| Brown | 40 | 20 | 40 |
| Black | 50 | 30 | 40 |

Test the hypothesis that there is an association between hair colour and eye colour at $5 \%$ level of significance.

## OR

The manager of a firm has received a complaint from the employees of the production department that their weekly compensation is less than the compensation of the marketing department. To verify this claim, the manager
has taken a random sample of 5 employees from the production department and 6 employees from the marketing department. The data collected are shown below :

| Weekly Compensation (in ₹) |  |
| :---: | :---: |
| Production <br> Department | Marketing <br> Department |
| 5000 | 5500 |
| 5200 | 5150 |
| 4800 | 4900 |
| 5300 | 5250 |
| 5100 | 4830 |
|  | 5020 |

In case the distribution of weekly compensation is not known, to examine whether the firm offers different compensation packages to the employees of the production and marketing departments, answer the following :
(i) Are both samples paired or independent? $\quad 1$
(ii) State the null and alternative hypotheses. 1
(iii) Which test is used for testing the null hypothesis in this case and why?
(iv) Conduct the suitable test at $5 \%$ level of significance and interpret the result.

