No. of Printed Pages : 4

MMTE-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00655

June, 2016

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Answer any **five** questions from questions no. 1 to 6. Use of calculator is **not** allowed.*

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1. (a) Explain what a simple communication channel is, with the help of a diagram. *2*

(b) Define the dual code of a code. Find the dual code of a code C generated by the matrix $G = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ over $\mathbf{F}_2$. Also find the generator matrix of the dual code of C. *5*

(c) Find the 2-cyclotomic cosets modulo 31. *3*

2. (a) Let r be an integer with $0 \leq r \leq m$. Let R(r, m) denote the r^{th} order RM code of length 2^m. Prove that $R(m, m)^{\perp} = \{0\}$, and $R(r, m)^{\perp} = R(m - r - 1, m)$ for $0 \leq r < m$. *5*

(b) Generate a field with 16 elements with the polynomial $x^4 + x + 1$. *5*

3. (a) Find the generating idempotent for a cyclic code C of length 7 over $\mathbf{F}_2$ with generator polynomial $1 + x + x^3$. *5*

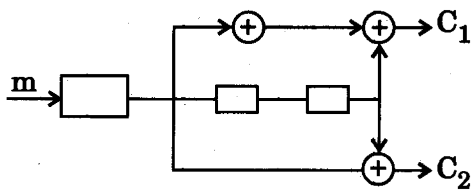
(b) Let C be a cyclic code in R_n and let e(x) be a non-zero idempotent in C. Prove that $C = \langle e(x) \rangle$ iff e(x) is the unity of C. *3*

(c) 'There is a unique self-dual code of length 7 over $\mathbf{F}_2$.' Is this statement true ? Give reasons for you answer. *2*

4. (a) Give an example of a BCH code, with justification. *3*

(b) Define a low density parity check code, and give an example. *3*

(c) Find the convolutional code for the message 1011. The convolutional encoder is given below : *4*



5. (a) Find the weight distribution and weight enumerator of the code C generated by the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \text{ over } \mathbf{F}_2.$$ *3*

(b) Let p be an odd prime and let a be in $\mathbf{Z}_p$ with $a \not\equiv 0 \pmod p$. If a is a square, then prove that the multiplicative order of a is a divisor of $\frac{(p-1)}{2}$. *3*

(c) Give the criteria for the existence of duadic codes of length n over $\mathbf{F}_2$ and $\mathbf{F}_3$. Also, find n, n > 10, such that duadic codes of length n exist over $\mathbf{F}_2$, $\mathbf{F}_3$. Justify your answer. *4*

6. (a) Show that the $\mathbf{Z}_4$-linear codes with generator matrices

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

are monomially equivalent. *2*

(b) Let C be a self-orthogonal $\mathbf{Z}_4$-linear code with $c \in C$. Prove that

$$wt_L(c) \equiv 0 \pmod 2$$

$$wt_E(c) \equiv 0 \pmod 4$$ *2*

(c) Let C be the [15, 7] narrow-sense binary BCH code with designed distance $\delta = 5$, which has defining set

$$T = \{1, 2, 3, 4, 6, 8, 9, 12\}.$$

Using the primitive 15^{th} root of unity α, $\alpha^4 = \alpha + 1$, the generator polynomial of C is

$$g(x) = 1 + x^4 + x^6 + x^7 + x^8.$$

If $y(x) = 1 + x + x^5 + x^6 + x^9 + x^{10}$ is received, find the transmitted code word. *6*
