No. of Printed Pages : 5

MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00375

Term-End Examination

June, 2016

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is* ***compulsory****. Answer any* ***four*** *questions out of questions no. 2 to 7. Use of calculators is* ***not*** *allowed.*

1. State whether the following statements are *true* or *false*. Justify your answers with the help of a short proof or a counter-example. *5×2=10*

(a) $\mathcal{L}^{-1}\left[ln\left(\dfrac{s+b}{s+a}\right)\right] = \dfrac{1}{t}(e^{-at} + e^{-bt}).$

(b) Euler's method is to be used to solve the initial value problem

$$y' = -50y, y(0) = 2.$$

The value of h that can be used so that the method produces stable results is $h < 0{\cdot}04$.

(c) The method

$$-\lambda\, u_{i-1}^{n+1} + (1+2\lambda)\, u_i^{n+1} - \lambda\, u_{i+1}^{n+1} = u_i^n,$$

for the solution of the equation $v_t = u_{xx}$ is unconditionally stable.

(d) $\int J_2(x)\,dx = 2J_1(x) - \int J_0(x)\,dx + c.$

(e) The p.d.e. $\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + (1+y^2)\frac{\partial^2 u}{\partial y^2}$ is parabolic in the region $x^2 - y^2 > 1$.

2. (a) Express $f(x) = e^{6x}$ as a Hermite series. Hence, evaluate

$$\int_{-\infty}^{\infty} e^{-x^2+6x}\, H_n(x)\,dx.$$ 5

(b) Derive the five-point formula for the Poisson equation $u_{xx} + u_{yy} = G(x, y)$ with uniform mesh h. Find the truncation error and the order of the method. 5

3. (a) Find the series solution about $x = 0$, of the differential equation

$$9x\,(1+x)\,y'' - 6y' + 2y = 0.$$ 5

(b) Find $y(0 \cdot 2)$ with $h = 0 \cdot 2$, for the initial value problem

$$y' = t^2 - y^2, \; y(0) = 2$$

using Runge-Kutta fourth order method. *3*

(c) If H_n is a Hermite polynomial of degree n, then show that $H_n' = 4n(n-1) H_{n-2}$. *2*

4. (a) Using Laplace transforms, solve the initial boundary value problem

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, \; 0 < x < \infty$$

$$y(0, t) = f(t), \; y(x, 0) = 0$$

$y(x, t)$ is bounded as $x \to \infty$, and

$$\frac{\partial y}{\partial t} = 0 \text{ at } t = 0.$$ *5*

(b) Solve the IVP $y' = -2xy^2$, $y(0) = 1$ with $h = 0 \cdot 2$ on the interval $[0, 0 \cdot 4]$ using the predictor-corrector method.

$$P : y_{k+1} = y_k + \frac{h}{2}(3y_k' - y_{k-1}')$$

$$C : y_{k+1} = y_k + \frac{h}{2}\left(y_{k+1}' + y_k'\right).$$

Perform two corrector iterations per step. Use second order Taylor series method with $h = 0 \cdot 2$ to obtain the starting value. *5*

5. (a) Show that the method

$$\delta_t^2 u_j^n = \frac{r^2}{2} \delta_x^2 \left[u_j^{n+1} + u_j^{n-1}\right]; \; r = \frac{k}{h}$$

for the numerical solution of the equation $u_{tt} = u_{xx}$ is of order $O(k^2 + h^2)$, where k and h are step lengths in time and space directions respectively. 5

(b) Solve the boundary value problem using the finite difference method with second order approximations with h = 1/3 : 5

$$x^2y'' + xy' - y = 0,$$

$$y(1) = 5, \; y(2) = 7.$$

6. (a) Find $\mathcal{F}^{-1}\left[\frac{1}{\alpha^2 + 16}\right]$, where $\mathcal{F}^{-1}$ is the inverse Fourier transform. 4

(b) Find the solution of the boundary value problem

$$u_{xx} + u_{yy} = x^2 + y^2$$

$$u(x, y) = x^2 - y^2 \text{ on the boundary}$$

$$0 \le x \le 4, \; 0 \le y \le 4, \; 0 \le x + y \le 4,$$

using the five-point formula. Assume h = 1. 6

7. (a) The solution of the boundary value problem

$$\nabla^2 u = 0, \ 0 \le x \le 1, \ 0 \le y \le 1$$

$$u = x + y,$$

on the boundary is being obtained by the Galerkin method with triangular elements and $h = \frac{1}{2}$. Find the contribution of any one of the element equations in $\frac{1}{2} \le x \le 1, \frac{1}{2} \le y \le 1.$ *6*

(b) If $f(x) = 0, \ -1 < x < 0$

$$= x, \ 0 < x < 1.$$

Show that

$$f(x) = \frac{1}{4} P_0(x) + \frac{1}{2} P_1(x) + \frac{5}{16} P_2(x) - \frac{3}{32} P_4(x) + \ldots$$

where $P_n(x)$ is a Legendre polynomial of degree n. *4*