M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

00496

Term-End Examination
June, 2016

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

(Weightage : 70%)

Note : *Question no. 1 is* ***compulsory****. Answer any* ***four*** *questions from questions no. 2 to 6. The use calculators is* ***not*** *allowed.*

1. Which of the following statements are *True*, and which are *False* ? Give reasons for your answers. *10*

   (a) There is a finite field of order 45.

   (b) A group G of order 9 has a representation of order 2.

   (c) Every free abelian group is a free group.

   (d) Every set is a semi-group.

   (e) If G is an infinite group acting on an infinite set X, then no element of X has finite stabiliser.

2. (a) Show that for any group G,

$$Z(G) = \bigcap_{x \in G} Z(x).$$ 3

(b) Find the degree of $\mathbf{Q}(\sqrt{5}, \sqrt{7})$ over $\mathbf{Q}$. Give reasons for each step. 4

(c) Give an example, with justification, of a phase structure grammar with alphabet $\mathbf{Z}_5$, and for which the set of grammar symbols is the underlying set of the centre of A_4. 3

3. (a) Prove that a group of order 28 has a normal subgroup of order 7. 3

(b) Prove that S_n is not simple, for all $n > 1$. 2

(c) Simultaneously solve : 5

$$x \equiv 2 \pmod 5$$

$$x \equiv 3 \pmod 7$$

$$x \equiv 4 \pmod{11}$$

4. Compute the character table of D_5. 10

5. (a) Prove that SO_2 is a subgroup of SU_2. Further, show that the subgroup of diagonal matrices in SU_2 is conjugate to SO_2. 4

(b) Let $K = \mathbf{Q}(\sqrt{-5}, \sqrt{-2})$ with Galois group $\{1, \rho, \sigma, \rho\sigma\}$, where ρ and σ are the $\mathbf{Q}$-automorphisms of K defined by

$$\rho(\sqrt{-5}) = -\sqrt{-5}, \ \rho(\sqrt{-2}) = -\sqrt{-2}$$

$$\sigma(\sqrt{-5}) = -\sqrt{-5}, \ \sigma(\sqrt{-2}) = \sqrt{-2}.$$

Find the fixed fields of the subgroups $< \rho >$ and $< \sigma >$. 4

(c) Give an example, with justification, of a semi-group which is not a group. 2

6. (a) Show that $SP_2(\mathbf{R})$ acts transitively on $\mathbf{R}^2 \setminus \{(0, 0)\}$. 4

(b) Find the elementary divisors, and the invariant factors, of the group

$$\mathbf{Z}_6 \times \mathbf{Z}_{22} \times \mathbf{Z}_{15} \times \mathbf{Z}_{121}.$$ 3

(c) Let $A = \begin{bmatrix} 9 & -7 \\ 13 & -10 \end{bmatrix}$ and $G = < A >$. Find a G-invariant form on $\mathbf{C}^2$. 3