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BME-001

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

00280

Term-End Examination

June, 2016

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of calculator is allowed.*

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1. Answer any ***five*** of the following : *5×4=20*

(a) Which of the following functions are onto ?

(i) $f : R \rightarrow R$ defined by $f(x) = 3x + 7$

(ii) $f : R^+ \rightarrow R$ defined by $f(x) = \sqrt{x}$

(iii) $f : R \rightarrow R$ defined by $f(x) = x^2 + 1$

(iv) $f : X \rightarrow R$ defined by $f(x) = \frac{1}{x}$, where X stands for the set of non-zero real numbers.

(b) Which of the following functions are *even, odd* or neither *even nor odd* ?

(i) $x \to x^2 + 1$, for all $x \in R$

(ii) $x \to x^2 - 1$, for all $x \in R$

(iii) $x \to \cos x$, for all $x \in R$

(iv) $x \to |x|$, for all $x \in R$

(v) $f(x) = \begin{cases} 0, \text{ if } x \text{ is rational} \\ 1, \text{ if } x \text{ is irrational} \end{cases}$

(c) Evaluate the limits that exist :

(i) $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{(x+2)^2}$

(ii) $\lim\limits_{x \to 3} \dfrac{(x^2 + x - 12)^2}{x - 3}$

(iii) $\lim\limits_{x \to 0} \dfrac{\tan x}{x}$

(iv) $\lim\limits_{x \to 0} \dfrac{\sqrt{3+x} - \sqrt{3}}{x}$

(d) Which of the following functions are continuous ?

(i) $f(x) = \begin{cases} 2x - 1, \text{ if } x < 0 \\ 2x + 1, \text{ if } x \geq 0 \end{cases}$

(ii) $f(x) = \begin{cases} \dfrac{x}{|x|}, \text{ if } x \neq 0 \\ 0, \text{ if } x = 0 \end{cases}$

(e) Find the derivatives of the functions

(i) $y = x^2$, and

(ii) $y = 3x^2 + 5x - 1$.

(f) Use the method of implicit differentiation to calculate $\frac{dy}{dx}$, if

$$2x^2 - 3xy + 4y^2 = 8.$$

2. Answer any ***four*** of the following : *4×5=20*

(a) If P(3, –2, 1) and Q(1, 2, –4) are the initial and terminal points respectively of a vector a, find the component of a and $|a|$.

(b) Show that the set of vectors

$5a + 6b + 7c$, $7a - 8b + 9c$, $3a + 20b + 5c$ are coplanar.

(c) A particle is acted on by constant forces $-3\hat{i} + 2\hat{j} + 5\hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$ and is displaced from the point (2, –1, –3) to the point (4, –3, 7). Find the total work done by the forces.

(d) A particle moves so that its position vector is given by

$$\vec{r} = \cos \omega t\,\hat{i} + \sin \omega t\,\hat{j}.$$

Show that the velocity $\vec{v}$ of the particle is perpendicular to r and $\vec{r} \times \vec{v}$ is a constant vector.

(e) Let C be the line segment from A(0, 0, 1) to B(1, 1) and let $f(x, y) = x + y^2$. Evaluate $\int_C f(x, y)\, ds$, when

(i) C is characterized by $x = t$, $y = t$, $0 \le t \le 1$.

(ii) C has parametric representation

$$x = \sin t,\ y = \sin t,\ 0 \le t \le \frac{\pi}{2}.$$

(f) (i) $\text{div}\,[(\vec{r} \times \vec{a}) \times \vec{b}] = -2(\vec{a} . \vec{b})$

(ii) $\text{grad}\,[\vec{r}, \vec{a}, \vec{b}] = \vec{a} \times \vec{b}$

3. Answer any ***five*** of the following : *5×3=15*

(a) Which of the following sets are linearly independent ?

(i) { (1, 1, 0), (1, 2, 0), (2, 1, 0 }

(ii) { (1, –1, 1, –1), (–1, –1, –1, 1), (–1, 1, 1, 1) }

(iii) { (2, 2, 2), (3, 1, 1), (1, 3, 3) }

(b) Which of the following transformations are linear ?

(i) $T(x, y, z) = (y, z)$

(ii) $T(x, y, z) = (x, 2x - 1, 2x + y, z + 2, 3x)$

(iii) $T(x, y, z) = (0, 2x, y)$

(iv) $T(x, y, z, t) = (z, x, t, y)$

(c) Prove that

$$A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

is an orthogonal matrix.

(d) If the given matrix A is orthogonal, find the values of x, y and z.

$$A = \frac{1}{3}\begin{vmatrix} x & 2 & 2 \\ 2 & 1 & y \\ 2 & z & 1 \end{vmatrix}$$

(e) Express $(A + B)^2$ as a matrix, where

$$A = \begin{bmatrix} 2 & 3 \\ 6 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & 7 \\ 4 & 0 \end{bmatrix}.$$ Also verify

$(A + B)^2 = A^2 + AB + BA + B^2$.

(f) Matrix A has x rows and x + 5 columns. Matrix B has y rows and 11 – y columns. Given a condition that both AB and BA exist. Find x and y.

(g) Solve the following system of linear equations, by Cramer's rule :

$$2x - y + 3z = 2$$

$$x + 3y - z = 11$$

$$2x - 2y + 5z = 3$$

4. Answer any ***three*** of the following : *3×5=15*

(a) An anti-aircraft gun can take a maximum of four shots on an enemy's plane moving away from it. The probabilities of hitting the plane at first, second, third and fourth shots are 0·4, 0·3, 0·2 and 0·1 respectively. Find the probability that the gun hits the plane.

(b) An urn contains 10 white and 15 blue balls. Two balls are drawn at random in succession. Find the probabilities of the two balls together by virtue of sampling.

(c) A random variable x has the following probability function :

Value of x	– 2	– 1	0	1	2	3
p(x)	0·1	k	0·2	2k	0·3	k

Find the value of k and further calculate the mean and variance.

(d) Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample out of 10 tools chosen at random

(i) exactly two will be defective, and

(ii) more than one will be defective

by using Poisson approximation.

(e) The mean life time of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. The company claims that the average life of the bulbs produced by it is 1600 hours. Using the level of significance of 0·05, examine the claim of the company.