No. of Printed Pages : 5

ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

00028

June, 2016

ET-101(A) : MATHEMATICS – I

Time : 3 hours | *Maximum Marks : 70*

Note : *All the questions are* ***compulsory****. Use of scientific calculator is permitted.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Find $\frac{dy}{dx}$, when

$$\sqrt{1-x^2} - \sqrt{1-y^2} = 4(x+y).$$

(b) Determine the values of 'a' and 'b' for which the function 'f' defined by

$$f(x) = \begin{cases} \sqrt{1-x} - ax, & \text{when } x < 1 \\ a^2x - b, & \text{when } x = 1 \\ 5x^2 - 4x, & \text{when } x > 1 \end{cases}$$

is continuous at x = 1.

(c) Evaluate the following limits if they exist :

(i) $\lim_{x \to 0} \left[\frac{x \tan^{-1} x}{1 - \cos x} \right]$

(ii) $\lim_{x \to 1} \left[\frac{\sqrt{x^2 + 3} - \sqrt{5x - 1}}{x^2 - 1} \right]$

(d) If $z = \cot^{-1}\left(\frac{y + x}{\sqrt{y} - \sqrt{x}} \right)$, prove that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} + \frac{1}{4} \sin 2z = 0.$$

(e) Find the semi-vertical angle of a right circular cone of maximum volume whose slant height is given.

(f) If $y = p \sin(ln\, x) + q \cos(ln\, x)$, prove that

$$x^2 y_2 + x y_1 + y = 0.$$

Hence find y_{n+2}, using Leibnitz's theorem.

2. Answer any ***four*** of the following : *4×4=16*

(a) Evaluate the following integrals :

(i) $\int_0^1 \frac{dx}{2e^x - 1}$

(ii) $\int_0^{\sqrt{3}/2} \frac{\tan(D \sin^{-1} x)}{\sqrt{1 - x^2}} dx$

(b) Evaluate the following integrals :

(i) $$\int e^{-x/2} \cdot \frac{\sqrt{1 - \sin x}}{1 + \cos x} \, dx$$

(ii) $$\int_1^2 \frac{dx}{x^2 \sqrt{x^2 - 1}}$$

(c) A curve is drawn to pass through the points given by the following table :

x	0·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0
y	2·5	2·0	2·4	2·7	2·8	3·0	2·6	2·1

Estimate the area bounded by the curve, axis of x and the lines x = 1 and x = 4, using Simpson's Rule.

(d) Solve the differential equation :

$$\left[\sin y + \frac{y}{x}(1 + x)\right] dx + [x + \ln x + x \cos y] \, dy = 0$$

(e) Find the total length of the curve, given by

$$x = a \cos^3 \theta; \quad y = a \sin^3 \theta.$$

3. Answer any ***four*** of the following : *4×4=16*

(a) Find curl (ϕ grad ϕ), where

$$\phi \equiv \phi(x, y, z) = xyz.$$

(b) Find the gradient and the unit normal vector to the level surface,

$$x^2 + y^2 - z^2 = 4 \text{ at } (2, 1, 1).$$

(c) Show that $\frac{1}{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ satisfies Laplace's equation.

(d) Find the total work done, when a force

$$\vec{F} = (2x + yz)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}$$

moves a particle along the curve, $x = \sin t$, $y = \cos t$, $z = 1$ from $t = 0$ to $t = \frac{\pi}{2}$.

(e) Evaluate $\iint_S \vec{F} \cdot \hat{n} \, dS$, where $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is that part of the surface of the sphere, $x^2 + y^2 + z^2 = 1$, which lies in the first octant.

4. Answer any ***three*** of the following : *3×6=18*

(a) Find the eigen values and the eigen vectors of the matrix

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}.$$

(b) Check whether

$$W = \{(a_1, a_2, a_3) \mid a_i \in \mathbf{R}, i = 1, 2, 3 \text{ and } 2a_1 - 3a_2 = a_3\}$$

is a subspace of $\mathbf{R}^3$? If so, find its dimension and a basis.

(c) Let T be the linear operator on $\mathbf{R}^3$ defined by $T(a_1, a_2, a_3) = (3a_1, a_1 - a_2, 2a_1 + a_2 + a_3)$.

Is T invertible ? If so, find a rule for T^{-1} like the one which defines T.

(d) Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3.$$