No. of Printed Pages : 3 **MMTE-005**

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00151

Term-End Examination

June, 2014

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

Weightage : 50%

Note : *Answer any* ***five*** *questions from question nos. 1 to 6. Calculators are* ***not*** *allowed.*

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1. (a) Define a Linear Code and give an example of the same. *3*

   (b) Prove that a self-dual code has even length n and dimension $\frac{n}{2}$. *3*

   (c) Let C be the Hamming code with the parity check matrix

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

   (i) Encode the message 0111.

   (ii) If the received codeword is 0110100, decode and find the message. *4*

2. (a) Define perfect code and give an example with justification. *1+2*

(b) Let $f(x) = x^2 + x + 1 \in \mathbf{F}_5[x]$.

(i) Prove that f(x) is irreducible over $\mathbf{F}_5$.

(ii) Let α be a root of f(x). Show that α is not primitive.

(iii) Find a primitive element of the form $a\alpha + b$, where $a, b \in \mathbf{F}_5$. *7*

3. (a) Prove that the integers modulo n do not form a field if n is not prime. *2*

(b) Find the 2 cyclotomic cosets modulo 7. Hence, factorize $x^7 - 1$ over $\mathbf{F}_2[x]$. *3*

(c) Prove that a self-orthogonal binary cyclic code is doubly-even. *5*

4. (a) Write down the Peterson – Gorenstein – Zierler Decoding Algorithm for BCH code. *4*

(b) State the MacWilliams equations. *2*

(c) Find the weight enumerator of the binary code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$ *4*

5. (a) Construct the generating idempotents of binary duadic codes of length 7. *4*

(b) Let C be the $\mathbf{Z}_4$-linear code of length 4 with generator matrix $G = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 3 & 2 \end{pmatrix}$.

List all codewords of C. *3*

(c) If C is a $\mathbf{Z}_4$-linear code, prove that the Gray image of C is distance invariant. *3*

6. (a) Define convolutional code and give an example. *3*

(b) Prove that $G = [1 + D^2 + D^3 \quad 1 + D^2]$ is a canonical generator matrix for a (2, 1) code. *4*

(c) Let C be the [7, 4, 2] binary code with parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Draw the Tanner graph for this code and mark variable nodes and check nodes. *3*