M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

⊓⊓226   ## M.Sc. (MACS)

Term-End Examination

June, 2014

MMTE-002 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 2 hours   *Maximum Marks : 50*

Note : *Do any **five** questions from questions 1 to 6. Use of calculator is **not** allowed.*

1. (a)   For the set of keys {1, 4, 5, 10, 16, 17, 21},
        draw binary search trees of height 3, 4, 5
        and 6.   *6*

    (b)   Show that a full parenthesization of an
        n-element expression with binary operators
        has exactly n – 1 pairs of parentheses.   *4*

2. (a)   Solve the following recurrence for $n = 2^k$ *3*

$$T(n) = \begin{cases} 2 & \text{if } n = 2 \\ 2T\left(\dfrac{n}{2}\right) + n & \text{if } n = 2k, \ k > 1 \end{cases}$$

(b)   Illustrate the execution of Counting Sort on the array A = [3, 6, 4, 1, 3, 4, 1, 4], noting that all elements are ≤ 6.   *3*

(c)   Describe Divide and Conquer algorithm for finding the closest pair of points in a finite subset of the plane.   *4*

3.   (a)   Determine the longest common subsequence of (1, 0, 0, 1, 0, 1, 0, 1) and (0, 1, 0, 1, 1, 0, 1, 1, 0). Illustrate the principle of optimality, while showing one step of recursion.   *5*

(b)   Find all the solutions to the equation $15x = 10 \pmod{35}$.   *5*

4.   (a)   What is the minimum and maximum number of elements in a heap of height h ? Illustrate the difference between max-heap property and binary-search-tree property through an example.   *5*

(b)   Illustrate the radix sort algorithm using the following array :   *5*

{3197, 4216, 3814, 5638, 7132, 8146, 5328, 7896, 4379, 6384, 9376}.

5. (a) Illustrate through the following example, why Dijkstra's algorithm for single source shortest path, may fail, in the presence of negative edge weights ? *5*
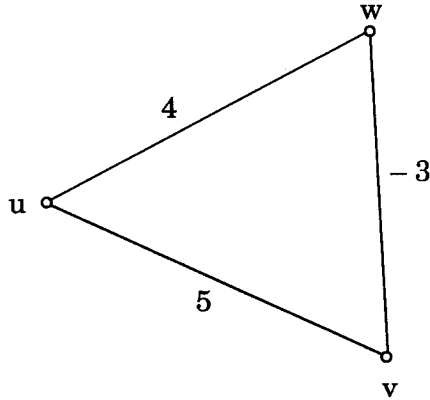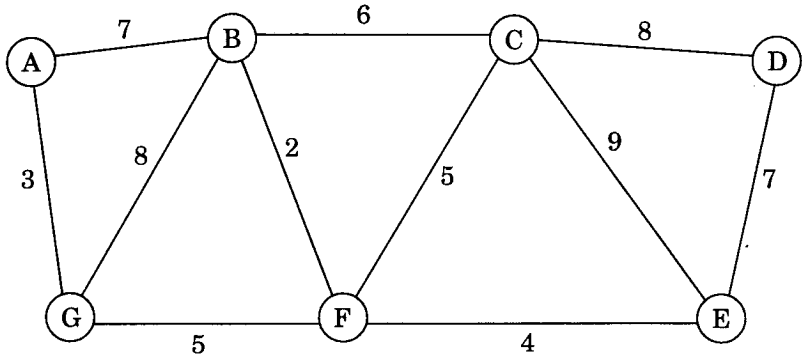

(b) Define Euler phi function and compute the value of Euler's phi function for 45, that is, compute $\phi(45)$. *3*

(c) Rank the following functions by order of growth : that is, find an ordering g_1, g_2 and g_3 of the functions satisfying $g_1 = O(g_2)$, $g_2 = O(g_3)$: $n!$, e^n, $n^{\lg \lg n}$. *2*

6. (a) Find a minimal spanning tree of the following graph using Kruskal's algorithm : 5

(b) Illustrate all steps of Rabin-Karp Miller algorithm for P = 1035, T = 140610304 and Q = 7. 5

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