No. of Printed Pages : 3

MMTE-001

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00116

Term-End Examination

June, 2014

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

Weightage : 50%

Note : *Question no.* **1** *is* ***compulsory****. Answer* ***any four*** *from the rest* ***six*** *(2-7). Calculators or any other electronic devices are* ***not*** *allowed.*

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1. State, giving justifications or illustrations, whether each of the following statements is **true** or **false** :

(a) A bipartite graph has no cycles as induced sub graphs. **5x2=10**

(b) Complement of a disconnected graph is always connected.

(c) If the minimum vertex degree $\delta(G) \geq 2$, then G contains a cycle.

(d) Number of even degree vertices in a graph is always odd.

(e) Every complete graph has a perfect matching.

2. (a) Draw a diagram of the Petersen graph. Determine the maximum size of a clique, the maximum size of an independent set and the maximum length of a cycle in the Petersen graph. **6**

(b) Prove that the center of a tree consists of a vertex or a pair of adjacent vertices. **4**

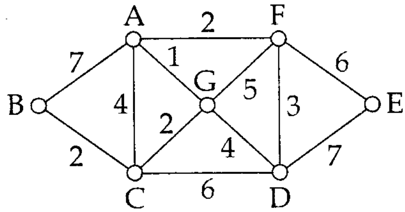
3. (a) Prove that a connected graph G is Eulerian if all its vertices are of even degree. 6

(b) Prove that every n-vertex graph with at least n edges contains a cycle. 4

4. (a) If G is a simple n-vertex graph with $\delta(G) \geq \frac{n-1}{2}$, prove that G is connected. Is the converse true ? Justify your answer. 6

(b) Draw a cubic graph G for which $\kappa(G) = 1$ and $\kappa'(G) = 2$. Justify your answer. 4

5. (a) Find a minimal spanning tree in the following graph using Prim's Algorithm : 4



(b) State and prove Hall's Marriage theorem. 6

6. (a) Describe the Greedy Algorithm for Graph Colouring. 2

(b) Use the Greedy Algorithm to find a proper colouring of vertices in the Petersen graph. 4

(c) Define Hamiltonian closure of a graph G and prove that Hamiltonian closure of a graph is well-defined. 4

7. (a) Draw the diagram of a connected plane graph G with 10 vertices and $\delta(G) = 3$. 3

(b) Draw the dual of the graph drawn in part 7(a). 3

(c) Prove that the chromatic number and the clique number are same for any interval graph. 4