No. of Printed Pages : 7

MMT-008

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00536

Term-End Examination

June, 2014

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 hours *Maximum Marks : 100*
(Weightage : 50%)

Note : *Question no. 8 is* ***compulsory****. Answer any* ***six*** *questions from question numbers 1 to 7. Use of calculator is* ***not*** *allowed.*

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1. (a) Joint p.m.f. of random vector (X, Y) is given in the following table :

X \ Y	0	1	2
0	1/8	0	2/8
1	0	1/8	1/8
2	1/8	2/8	0

Find :

(i) marginal p.m.f. of X,

(ii) $P[1 \leq X \leq 2]$ and

(iii) conditional p.m.f. of Y given X = 1. *6*

(b) A post office has two counters. The first handles money-orders, registry and speed post and the second handles all other business. The service times of both the counters follow exponential process with mean 2 minutes. Customers arrive at the first counter 20 per hour and at the second counter 25 per hour in Poisson process. Find the time spent in the post office if both the counters start to handle all the customers. *9*

2. (a) Let $\{X_n, n \geq 1\}$ be iid renewal periods with common p.m.f. $P(X_n = 0) = 0{\cdot}4$, $P(X_n = 1) = 0{\cdot}4$ and $P(X_n = 2) = 0{\cdot}2$. Obtain Laplace transform of renewal function $M(t)$. *4*

(b) Explain birth and death process, infinitesimal transition rates and generator with the help of one example of each. In a birth and death process $\lambda_k = \lambda$, $\mu_k = \mu$, for all $k \geq 0$, $\lambda, \mu > 0$. Under what condition does stationary distribution exist ? Find the stationary distribution also. *9*

(c) Write two advantages and two disadvantages of conjoint analysis. *2*

3. (a) For the Markov chain having following transition matrix :

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 3/4 & 0 & 1/4 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

find

(i) whether the chain is irreducible ? Give reasons.

(ii) probabilities of ultimate return to the states.

(iii) mean recurrence times to the states. 7

(b) Customers arrive at a ticket counter according to the Poisson process on average 2 every 5 minutes. Service time at the counter follows exponential distribution with mean 2 minutes.

Find :

(i) proportion of time a customer finds empty counter.

(ii) average number of customers at the counter.

(iii) probability that a new customer will have to wait more than 2 minutes at the counter. 8

4. (a) Explain branching process. In a branching process $\{X_n, n \geq 0\}$ starting from single individual, each individual generates offsprings according to Poisson law. Find p.g.f. of X_n. 7

(b) Explain renewal process and show that a Poisson process is a renewal process also. If the renewal periods in a renewal process are iid exponential then find the distribution of renewal sequence. *8*

5. (a) For n = 15 pair of observations from a bivariate normal population, the following summary statistics are obtained :

$$\overline{X} = \begin{bmatrix} 4\cdot5 \\ 8\cdot0 \end{bmatrix} \quad S = \begin{bmatrix} 0\cdot5 & 0\cdot3 \\ 0\cdot3 & 1\cdot7 \end{bmatrix} \quad S^{-1} = \begin{bmatrix} 2\cdot237 & -0\cdot395 \\ -0\cdot395 & 0\cdot658 \end{bmatrix}$$

Test the hypothesis $\overline{\mu}' = [4, 9]$ at 5% level of significance.

[You may like to use the following values $F_{2,\,13}(0\cdot05) = 3\cdot806$), $F_{2,\,14}(0\cdot05) = 3\cdot74$] *8*

(b) Let $\mathbf{y} \sim N_3(\overline{\mu}, \Sigma)$ where

$$\mu = \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix}, \quad \mathbf{\Sigma} = \begin{pmatrix} 2 & 2 & -1 \\ 2 & 5 & 1 \\ -1 & 1 & 2 \end{pmatrix}$$

Obtain :

(i) distribution of $\mathbf{Cy}$ where

$$\mathbf{C} = \begin{pmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \end{pmatrix}.$$

(ii) linear combination $z = \boldsymbol{l}'\mathbf{y}$ such that $z \sim N(0, 1)$. *7*

6. (a) Obtain spectral decomposition of

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$ and evaluate A^4 using the result. 7

(b) Let $\mathbf{y} \sim N_4(\bar{\mu}, \Sigma)$ where $\mu' = (-3 \;\; 2 \;\; -1 \;\; 4)$

and $$\mathbf{\Sigma} = \begin{pmatrix} 3 & -3 & -1 & 2 \\ -3 & 6 & -1 & -2 \\ -1 & -1 & 2 & 0 \\ 2 & -2 & 0 & 4 \end{pmatrix}$$

Obtain :

(i) marginal distribution of $\begin{pmatrix} y_1 \\ y_3 \end{pmatrix}$

(ii) conditional distribution of $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ given $\begin{pmatrix} y_3 \\ y_4 \end{pmatrix}$

(iii) $r_{12}, r_{12.34}$. 8

7. (a) Obtain triangular square root of 7

$$A = \begin{bmatrix} 9 & 3 & 3 \\ 3 & 5 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

(b) The variance covariance matrix of three variables X_1, X_2 and X_3 is

$$\Sigma = \begin{pmatrix} 104 & 193 & 105 \\ 193 & 413 & 192 \\ 105 & 192 & 107 \end{pmatrix}$$

The eigenvalues and corresponding eigenvectors are given below :

$\lambda_1 = 602{\cdot}2 \quad \bar{a}_1' = (0{\cdot}40, 0{\cdot}82, 0{\cdot}40)$

$\lambda_2 = 21{\cdot}4 \quad \bar{a}_2' = (0{\cdot}52, -0{\cdot}57, 0{\cdot}63)$

$\lambda_3 = 0{\cdot}4 \quad \bar{a}_3' = (-0{\cdot}75, 0{\cdot}04, 0{\cdot}66)$

(i) Obtain principal components.

(ii) Obtain variances of principal components.

(iii) Verify total variances explained by principal components is equal to total variances of original variables.

(iv) Obtain proportion of variation explained by first two components. *8*

8. State whether following statements are *true* or *false*. Justify your answer. *2×5=10*

(i) Two events A and B are non-null and mutually exclusive, then $P(A \mid B) = 0$.

(ii) A state i in a Markov chain is persistent, then the series $\sum_{n=0}^{\infty} p_{ii}^{(n)}$ is divergent.

(iii) If P(s) be the p.g.f. of a random variable then P(1) will be its mean.

(iv) When n is large then

$n(\overline{X} - \overline{\mu})' S^{-1} (\overline{X} - \overline{\mu})$ follows

approximately χ_p^2.

(v) Canonical correlation is a particular case of multiple correlation.