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MMT-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00775

Term-End Examination

June, 2014

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

Note : *Question 1 is* ***compulsory****. Attempt any* ***three*** *other questions. Use of calculators is* ***not*** *allowed.*

1. State giving reasons whether the following statements are *true* or *false* : *2×5=10*

   (a) The hyperbola $x^2 - y^2 = 1$ under the mapping $w = -z^2$ is mapped into the straight line $u = 1$.

   (b) If $f(z)$ is differentiable at z_0 then $|f(z)|$ is also differentiable at z_0.

   (c) All the singular points of $f(z) = \cot z$ are isolated.

   (d) $f(z) = \sin z$ is bounded.

(e) If C_R denotes any positively oriented circle with centre at the origin and of radius R, then $\lim_{R \to \infty} \int_{C_R} \frac{\text{Log } z}{z^2} \, dz = 0.$

2. (a) If f(z) is analytic in a domain D such that f(z) is real for each $Z \in D$, then prove that f(z) must be a constant function. *2*

(b) Let $T(z) = \frac{2z+1}{2+z}$. Show that whenever $|z| \leq 1$, then $|T(z)| \leq 1$. Find fixed points of T(z). *3*

3. (a) It is given that f(z) is an entire function, f(z) has a zero of order at least 3 at z = 0 such that $|f(z)| \leq 2|z|^3$ for all z. Prove that f(z) is a polynomial of degree at most 3. *3*

(b) Find all the roots of the equation cosh z = 3. *2*

4. (a) Let f(z) = u(x, y) + iv(x, y) be a function continuous on a closed and bounded region R and analytic in the interior of R. Suppose f(z) is non-constant function in the interior of R. Then apply the maximum modulus principle to $e^{if(z)}$ to prove that the minimum of v(x, y) occurs on the boundary of R and never in the interior. *3*

(b) Let C be the positively oriented circle $|z| = 2$. If $g(w) = \int_C \frac{z^2 - 4z + 1}{z - w} dz$, $(|w| \neq 2)$, then find $g(1)$. Further, prove that $g(w) = 0$ for $|w| > 2$. 2

5. Evaluate $\int_0^{2\pi} \frac{d\theta}{2 - \sin\theta}$. 5