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MMT-003

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

June, 2014

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

(Weightage 70%)

Note : *Question no. 1 is* ***compulsory****. Do any* ***four*** *questions from questions 2 to 6. Calculators are* ***not*** *allowed.*

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1. State which of the following statements are *true* and which are *false*. Give reasons for your answer. *2×5=10*

(i) There is a non-trivial group homomorphism from a cyclic group of order 10 to S_5.

(ii) All the irreducible polynomials over **R** are quadratic.

(iii) The group SO_2 (**C**) is bounded.

(iv) For a character χ of degree $d > 1$ of a finite group G, there is an element $g \in G$ with $|\chi(g)| = d^2$.

(v) In any finite abelian group all Sylow subgroups are cyclic.

2. (a) Determine the number of elements of order 13 in a group of order 156. *3*

(b) The matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ has order 3 and therefore, it defines a matrix representation $\{1, A, A^2\}$ of the cyclic group G of order 3. Find a G-invariant form on $\mathbf{C}^2$. *4*

(c) Show that any subsemigroup of a finite group G is a subgroup of G. Is this statement true for subsemigroups in any infinite group ? *3*

3. (a) Find the number of abelian groups of order 392. Find the invariant factors of any one of the possible abelian non-cyclic groups of order 392. *5*

(b) Write down the conjugary classes of D_5. Interpreting D_5 as a symmetry group of a regular pentagon, find the stabiliser of a vertex in a regular pentagon. *5*

5. Which of the following statements are true, and which are not ? Give reasons for your answers. *10*

(i) Eigenvectors corresponding to the same eigenvalues of a matrix are always linearly dependent.

(ii) If all the eigenvalues of a matrix A are zero, then A is similar to the zero matrix.

(iii) The sum of two normal matrices of order n is normal.

(iv) If A is a matrix with determinant 1, then A is a unitary matrix.

(v) If the characteristic polynomial of a matrix is $(x - 3)^2 (x - 2)^2$, then its minimal polynomial can be $(x - 3)^2$.