BME-001

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

01600

Term-End Examination

June, 2014

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of calculator is allowed.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Let a function f be defined as

$$f(x) = \begin{cases} x, & \text{if } 0 \le x < \frac{1}{2} \\ 0 & \text{if } x = \frac{1}{2} \\ x - 1 & \text{if } \frac{1}{2} < x \le 1. \end{cases}$$

Discuss the existence of $\lim\limits_{x \to \frac{1}{2}} f(x)$.

(b) Determine the values of a, b and c for which the function

$$f(x) = \begin{cases} \dfrac{\sin (a+1)\, x + \sin x}{x}, & \text{for } x < 0 \\ 0, & \text{for } x = 0 \\ \dfrac{(x + bx^2)^{+1/2} - x^{1/2}}{bx^{3/2}}, & \text{for } x > 0 \end{cases}$$

is continuous at $x = 0$.

(c) Show that the function

$$f(x) = 2x - \tan^{-1} x - \log \left(x + \sqrt{1 + x^2} \right)$$

continually increases as x changes from 0 to ∞.

(d) If $y = e^x \tan^{-1} x$, show that

$$(1 + x^2) \frac{d^2y}{dx^2} - 2\,(1 - x + x^2) \frac{dy}{dx} + (1 - x)^2\, y = 0$$

(e) Expand e^x in powers of $(x - 1)$, using Taylor's formula.

(f) Find the area bounded by

$$y = \begin{cases} 2x + 3, & \text{if } x \le 3 \\ -x + 12, & \text{if } x > 3 \end{cases}$$

and $x = 2$ and $x = 5$.

(g) Solve any ***one*** of the following :

(i) $\dfrac{dy}{dx} = \dfrac{4x + 6y + 5}{2x + 3y + 4}$

(ii) $\dfrac{dy}{dx} + \dfrac{x}{1 - x^2} y = x\sqrt{y}$

2. Answer any ***four*** of the following : *4×4=16*

(a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are perpendicular to each other, show that

$$[\vec{a} \cdot (\vec{b} \times \vec{c})]^2 = a^2 b^2 c^2.$$

(b) A force of 78 gm acts at the point (2, 3, 5), the direction cosines of its line of action being 2 : 2 : 1. Find the magnitude of its moment about the line joining the origin to the point (12, 3, 4).

(c) Show that

$$\text{Curl } \frac{\vec{a} \times \vec{r}}{r^3} = -\frac{\vec{a}}{r^3} + \frac{3r}{r^5}(\vec{a} \cdot \vec{r})$$

(d) Show that the vector function

$$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$$

is irrotational and find the corresponding scalar function ϕ such that $\vec{F} = \nabla\phi$.

(e) Apply Stokes' Theorem to prove that

$$\int_C (y\,dx + z\,dy + x\,dz) = -2\sqrt{2}\,\pi a^2,$$

where C is the curve given by

$x^2 + y^2 + z^2 - 2ax - 2ay = 0,\ x + y = 2a$

and begins at the point (2a, 0, 0) and goes at first below the z-plane.

(f) Evaluate

$$\int_S \frac{\vec{r}}{r^3} \cdot d\vec{a},$$

where S denotes the sphere of radius a with centre at the origin.

(g) If $\vec{OA} = a\,\hat{i}$, $\vec{OB} = a\,\hat{j}$, $\vec{OC} = a\,\hat{k}$ are three co-terminous edges of a cube and S denotes the surface of the cube, evaluate

$$\int_S \{(x^2 - yz)\,\hat{i} - 2x^2y\,\hat{j} + 2\,\hat{k}\} \cdot \hat{n}\,dS$$

by expressing it as a volume integral.

(h) Show that grad $[\vec{r}, \vec{a}, \vec{b}] = \vec{a} \times \vec{b}$.

3. Answer any *six* of the following : *6×3=18*

(a) Use Gram-Schmidt process on basis

$$\{(0, 0, 1), (0, 1, 0), (1, 1, 1)\}$$

to obtain an orthonormal basis of R^3.

(b) Evaluate

$$A^2 - 4A + 5I,$$

if I is the unit matrix of order 3 and

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 3 & 2 \end{bmatrix}.$$

(c) If A^θ represents transpose of conjugate of matrix A of order 3 given by

$$A = \begin{bmatrix} 2 & 4-i & 3+2i \\ 5 & 3+i & 6+7i \\ 4-2i & 7 & 5+i \end{bmatrix},$$

show that $(A - A^\theta)$ is skew-Hermitian.

(d) Using adjoint method, find the inverse of the matrix

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$$

(e) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

(f) Test the following system of equations for consistency :

$$x - 4y + 7z = 14$$

$$3x + 8y - 2z = 13$$

$$7x - 8y + 26z = 5$$

(g) Obtain the characteristic equation and characteristic roots of the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

(h) Using Cramer's rule, solve the system of equations

$$x + y + z = 7$$

$$x + 2y + 3z = 16$$

$$x + 3y + 4z = 22$$

4. Answer any ***four*** of the following : *4×4=16*

(a) Calculate the expected frequencies from the table given below and use Chi-square test at 5% level of significance to state whether the two attributes are independent :

		Conditions of home	
		Clean	Dirty
Conditions of child as independent	Clean	70	50
	Fairly Clean	80	20
	Dirty	35	45

(Given χ^2 at 5% for 2 d.o.f. = 5·991,

for 3 d.o.f. = 7·815

and for 4 d.o.f. = 9·488.)

(b) For a given sample of 200 items drawn from a large population, the mean is 65 and standard deviation is 8. Find the 95% confidence limit for the population mean.

(c) For a normal distribution with mean 2 and variance 9, find the value of x such that the probability of interval [2, x] is 0·4115.

(From Normal Table,

$$\frac{1}{3\sqrt{2\pi}} \int_0^t e^{\frac{1}{2}t^2} \, dt = 0{\cdot}4115 \Rightarrow t = 1{\cdot}35)$$

(d) If x is a Poisson variate such that

$$P(x = 2) = 9P(x = 4) + 90\,P(x = 6),$$

find the mean of x.

(e) Using Binomial distribution, find the probability of rolling at the most 2 sixes in 5 rolls of a dice.

(f) The contents of three urns A_1, A_2 and A_3 are as follows :

A_1 : 1 white + 2 red + 3 black balls

A_2 : 2 white + 3 red + 1 black balls

A_3 : 3 white + 1 red + 2 black balls

An urn is chosen at random and two balls are drawn from it, which happen to be one red and one white. Calculate the probability that they came from urn A_2.