BIEEE-015

B.Tech. – VIEP – ELECTRICAL ENGINEERING (BTELVI)

Term-End Examination

00457

June, 2014

BIEEE-015 : STOCHASTIC CONTROL SYSTEMS

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any **five** questions. All questions carry equal marks.*

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1. (a) When is a random variable said to be a Gaussian random variable ? 4

   (b) Define the probability distribution and density function for a random variable X.
   Find the mean value of the function $g(X) = 4X^2$, where X is a random variable having the probability density function as

$$f_X(x) = \begin{cases} \frac{1}{2}\cos x & -\pi/2 < x < \pi/2 \\ 0 & \text{elsewhere} \end{cases}$$ 10

2. (a) Define joint characteristic function for two random variables X and Y. 4

(b) Random variables X and Y have the joint density function

$$f_{XY}(x, y) = \begin{cases} (x + y)^2 & -1 < x < 1,\ -3 < y < 3 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Find all the second order moments of X and Y.

(ii) What are the variances of X and Y ?

(iii) What is the correlation coefficient ? *10*

3. Define power spectral density.

Show that the power spectral density of a wide sense stationary discrete time stochastic process X(t) is given by

$$S(\omega) = \sum_{i=-\infty}^{\infty} r(L)\, e^{-j\omega L}$$

where r(L) is the autocorrelation function. *14*

4. (a) Define correlation, covariance and orthogonality for two random variables X and Y. Then define random variables V and W by V = X + a.Y and W = X – a.Y where a is a real number. *7*

(b) Two random variables X and Y are related by the expression Y = aX + b, where a and b are any real numbers.

Show that their correlation coefficient is

$$\rho = \begin{cases} 1 & \text{if } a > 0 \text{ for any b} \\ -1 & \text{if } a < 0 \text{ for any b} \end{cases}$$

and that their covariance is $C_{XY} = a\sigma_X^2$ where σ_X^2 is the variance of X.

5\. (a) Show that $\text{cov}\left[\tilde{X}(t \mid t_1), \hat{X}(t \mid t_1)\right] = 0.$ *7*

(b) Show that

$$\lambda(t \mid t_1) = -P^{-1}(t)[\hat{X}(t \mid t_1) - \hat{X}(t)]$$

is a solution to the fixed interval smoothing equations.

Hence derive the smoothing error variance equation. *7*

6\. (a) Discuss the significance of expected value operator and list its properties. *5*

(b) Discuss the role of Kalman filter in control system analysis and design. Also explain the algorithm of a Discrete Kalman filter. *9*

7\. (a) Draw and explain the structure of a multistage lattice filter. *7*

(b) Explain the properties of correlation matrix of a stationary discrete time stochastic process. *7*

8\. Write short notes on any ***two*** of the following : *2×7=14*

(a) Joint and Conditional probability

(b) Gauss – Markov process model

(c) Optimal fixed-lag smoothing

(d) LQR optimal controls