BICS-008

B.Tech. – VIEP – COMPUTER SCIENCE AND ENGINEERING (BTCSVI)

00174

Term-End Examination

June, 2014

BICS-008 : DISCRETE MATHEMATICAL STRUCTURES

Time : 3 hours *Maximum Marks : 70*

*Note : Part A is **compulsory** and carries 14 marks. Answer any **four** questions from Part B which carries 56 marks.*

PART A

1. Prove the following using Venn Diagram : 2

   $A \cap B \oplus C = (A \cap B) \oplus (A \cap C)$

2. Express the statement "Some students of Chemical Engineering are good in Mathematics" in symbolic form and write its negation. 2
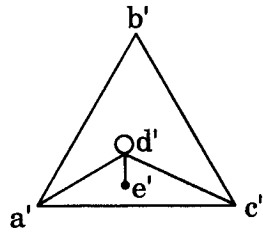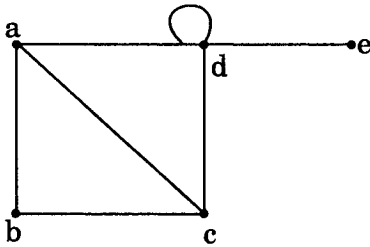
3. Find the inverse of the function $f(x) = 4e^{(6x+2)}$. 2

4. Show that the proposition 2

   $(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)$

   is a tautology.

5. If S denotes the set of positive integers ≤ 100 for $x, y \in S$, define $x * y = \min \{x, y\}$. Verify whether $(S, *)$ is a monoid assuming that $*$ is associative. **2**

6. State pigeon-hole principle. **2**

7. Are the following graphs isomorphic ? Give reason. **2**

PART B

Attempt any four questions.

8. (a) Prove by mathematical induction that for $n \geq 1$:

$$1.1! + 2.2! + 3.3! + \ldots\ldots + n.n! = (n + 1)!$$

(b) Let a and b be positive integers and suppose Q is defined recursively as follows :

$$Q(a, b) = \begin{cases} 0, & \text{if } a < b \\ Q(a-b, b) + 1 & \text{if } b \leq a \end{cases}$$

Find Q(2, 5), Q(12, 5), Q(5861, 7).   *7+7=14*

9. (a) Show that the set of the inverses of the elements of a right coset is a left coset i.e. $(Ha)^{-1} = a^{-1}H$.

(b) Prove that the set {1, 3, 4, 5, 9} is an abelian group under multiplication modulo 11 as composition.   *7+7=14*

10. (a) Minimize the following Boolean Expression and give its DNF

$$F(A, B, C, D) = \pi(0, 1, 4, 5, 8, 12, 13, 14, 15)$$

(b) Consider the set A = {2, 7, 14, 28, 56, 84} and the relation a $\leq$ b if and only if a divides b. Give the Hasse diagram for the poset (A, $\leq$).   *7+7=14*

11. (a) Without using truth table, show that

$$(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \leftrightarrow R$$

(b) Determine the validity of the following argument using propositional logic :

"If two sides of a triangle are equal, then their opposite angles are equal. Two sides of a triangle are not equal. Therefore, the opposite angles are not equal." *7+7=14*

12. (a) What are planar graphs ? Prove that Kuratowski's first graph (K_5) is non-planar.

(b) Solve the recurrence relation :

$$a_n - 7a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0 \text{ for } n \geq 2$$

5+9=14

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