(BTCSVI / BTECVI / BTELVI) B.Tech. (Degree)

00164

Term-End Examination

June, 2014

BICE-007 : MATHEMATICS-III

Time : 3 hours   *Maximum Marks : 70*

Note :   *(i)*   *All questions carry **equal** marks.*

       (ii)   *Attempt **any seven** questions.*

       (iii)   *Use of statistical table is **permitted**.*

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1. (a) Let $f(z)$ be analytic in a simply connected   5
   domain D. Then show that for any point z_0
   in D and any simple closed path C in D that
   encloses z_0

   $$\oint_c \frac{f(z)}{z - z_0}\, dz = 2\pi i\, f(z_0)$$

   (b) Show that for the function   5

   $$f(z) = \begin{cases} \dfrac{z^5}{|z|^4} & \text{for } z \neq 0 \\ 0 & \text{for } z = 0 \end{cases}$$

   The Cauchy-Riemann equation are satisfied
   at $z = 0$. But $f'(z)$ not exist at $z = 0$.

2. (a) Show that $\displaystyle\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta} = 2\pi$ 5

   (b) Evaluate $\displaystyle\oint_c \frac{\tan z}{z^2 - 1}\, dz$ where c is the circle,   5

   $$|z| = \frac{3}{2}$$

3. (a) A box contains 10 screws, three of which   5
        are defective. Two screws are drawn at
        random. Find the probability that none of
        the two screws is defective.

    (b) The first four moments of a distribution   5
        about the value '4', of the variable are -1.5,
        17, -30 and 108. State whether the
        distribution is leptokurtic or platy kurtic.

4. A basketball coach is attempting to evaluate the   10
    impact of an expensive player on the team's
    won/loss record. He considers the following
    table, which give number of games per season in
    which the player has seen at least twenty five
    minutes of floor time, together with wins over the
    season.

xi : (games)	62	58	59	73	69	75	72	81	46	79
yi : (wins)	47	46	47	56	55	62	55	63	38	64

    (a) Show that there is a significant linear
        correlation between games in which the
        player participated and team wins.
    (b) Write the equation of the regression line.

5. (a) Determine a 95% confidence interval for the   5
        mean μ of a normal population with
        variance $\sigma^2 = 16$, using a sample of size 200
        with mean 74.81.

    (b) In a production of iron rods let the diameter   5
        X be normally distributed with mean 2 inch
        and S. D(σ)0.008 inch. What percentage of
        defective can we expect if we set tolerance
        limits at 2 ± 0.02 inch.

6. (a) Find formulas for UCL, CL, LCL 5
   (corresponding to 3σ - limits) in the case of
   a control chart for the number of defectives,
   assuming that in a state of statistical control
   the fraction of defective is p.

   (b) A die is thrown 60 times and the results of 5
   these throws are given below :

No. of appeared on the die	1	2	3	4	5	6
Frequency	10	13	9	11	9	8

   claim on a 5% level that die is fair ?
   $\{\chi^2 (\alpha)$ at 5%, 5 degree of freedom $= 11.07\}$.

7. (a) Compute the rate of convergence of 5
   Newton-Raphson method.

   (b) Compute $\ln 9.2$ from $\ln 9.0 = 2.1972$, $\ln$ 5
   $9.5 = 2.2513$ by Lagrange interpolation and
   determine the error from $\ln 9.2 = 2.2192$
   upto (4 decimal).

8. Compute a 7 decimal-value of the Bessel function 10
   $J_0 (x)$ for $x = 1.72$ from the four values in the
   following table, using

   (a) Newton's Forward formula.

   (b) Newton's Backward formula.

x	0	1	2	3
$J_0(x)$	1.7	1.8	1.9	2.0

9. Apply the Runge-Kutta method to the following initial value problem, choosing $h = 0.2$ and computing four decimal. **10**

$$\frac{dy}{dx} = f(x, y) = x + y, \ y(0) = 0.$$

Find $y(.2)$.

10. Write short note on **any two** of the following :
    (a) Simpson's three-eighth rule. **5x2=10**
    (b) Gauss-Seidel method.
    (c) Testing of Hypotheses.