1. (a) What is big O notation? How is it different from \( \Omega \) notation?  
(b) Give an analysis of merge-sort. For simplicity, assume that the number of elements i.e. \( n \) is an exact power of two.  
(c) Explain limitations of Strassen's Algorithm for matrix multiplication.  
(d) Use Master's method to find tight asymptotic bounds for the following recurrence:  
\[ T(n) = T\left(\frac{n}{2}\right) + n \]  
(e) Give a divide and conquer based algorithm to find \( i^{th} \) largest element in an array of size \( n \).  
(f) What is regular expression? Write a regular expression over \( \Sigma = \{a, b\} \) to generate all string that start with a and end with two b's.
(g) Write binary search algorithm and evaluate its time complexity in the best, average and worst cases.

(h) Explain NP-complete problem with the help of an example.

2. (a) Find the topological ordering of the following graph:

```
A -> B -> C <- D
  |
  v
E <- F
```

(b) Write Kruskal’s algorithm and determine its time complexity.

(c) Represent the following graph using (i) Array; and (ii) Adjacency List

```
A -- 2 -- B -- 4 -- D
|     |     |     |
V     V     V     V
6      5     3     2

C -- 4 -- A
|     |
V     v
4      3

E -- 2
```

3. (a) Sort the given list using bubble sort and show the steps involved in the process: 2, 7, 5, 10, 21, 3

(b) Write Euclid’s algorithm for finding Greatest Common Divisor (G.C.D) of two natural numbers m and n.

(c) What is the benefit of preconditioning a problem space? Explain using an example.
(d) Consider the CFG:

\[ S \rightarrow SS \mid X a X a X \mid \wedge \]

\[ X \rightarrow bX \mid \wedge \]

Explain the language generated by this CFG.

4. (a) What is Push Down Automata? How is it different from Finite Automata.

(b) What is MinMax Algorithm? Explain how Alpha-Beta pruning helps in improving MinMax algorithm.

(c) What is best case analysis? Perform the best case analysis for Quick Sort.

5. (a) Explain each of the following, with an appropriate example:
   (i) NIM/Marienbad Game
   (ii) Principle of Mathematical Induction
   (iii) Halting Problem

(b) Trace how Depth First Search Traverses the following tree, when starting at node A.

```
    A
   / \  /  \\ 
  B   C  D  E  
 \   / \  /  \\  
 F   G  H   
```

MCS-031