No. of Printed Pages : **4**

MMTE-005

00232

M.Sc. MASTER IN MATHEMATICS WITH APPLICATIONS TO COMPUTER SCIENCE (MACS)

Term-End Examination

June, 2013

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage 50%)

Note : *(i) Answer* ***any five*** *questions from questions* ***1*** *to* ***6.***

(ii) Calculators are ***not*** *allowed.*

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1. (a) Explain the following terms with the help of an example. **3**

(i) Linear block code

(ii) Generator matrix

(iii) Parity check matrix

(b) Find the dual code of the binary code generated by the matrix. **4**

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(c) Let r be an integer with $0 \le r \le m$. Prove that the dimension of R (r,m) is $mc_0 + mc_1 + ... + mc_r$. **3**

2. (a) Explain the binary symmetric channel with the help of a diagram. 3

(b) Construct a finite field with 8 elements. 5

(c) Find the parity check matrix of a Hamming Code C whose generator matrix is given below : 2

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

3. (a) Prove that a self orthogonal binary cyclic code is doubly even. 5

(b) Define BCH code and show that the BCH code of design distance δ has minimum weight at least δ. 3

(c) Define quadratic residue code. 2

4. (a) For the binary code with the following generator matrix 'G' find the weight distribution polynomial. 5

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(b) Let f(x) and g(x) be polynomials in $Z_4[x]$. Prove that f(x) and g(x) are coprime iff $\mu[f(x)]$ and $\mu(g(x))$ are coprime in $F_2[x]$, where $\mu: Z_4(x) \to Z_2(x)$ is defined by $\mu(f(x)) = f(x) \pmod 2$ **5**

5. (a) Let C be a (4,2) convolutional code with generator matrix 'G' **6**

$$G = \begin{bmatrix} 1 & 1+D+D^2 & 1+D^2 & 1+D \\ 0 & 1+D & D & 1 \end{bmatrix}$$

Use elementary row operations to find '2' more generator matrices for C.

(b) Obtain the Tanner graph of the code whose parity check matrix is given below : **4**

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

6. (a) Let *C* be the Z_4-linear code of length 3 with generator matrix **4**

$$G = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

(i) List all codewords in *C*.

(ii) List all the codewords in the gray image of *C*.

(b) Find the generator polynomial and the minimum distance of Rs. [15,11] code. **4**

(c) Let C be a (10,2,4) LDPC code with parity check matrix **2**

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The received code vector is $(000011)\ 000^T$ Decode the message.
