No. of Printed Pages : **4**

MMTE-001

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

June, 2013

MMTE-001 : GRAPH THEORY

Time : 2 hours | *Maximum Marks : 50*
Weightage : 50%

Note : *Question no.* **1** *is* ***compulsory****. Answer* ***any four*** *out of the remaining six (2 to 7). Calculating devices are* ***not*** *allowed.*

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1. State whether the following statements are true or false ? Justify your answer with appropriate arguments or illustrations (**2** marks each) : **2x5=10**

(a) Complement of a disconnected graph is always connected.

(b) The Petersen graph is bipartite.

(c) In any group of persons, there are at least two with the same number of friends.

(d) Every Eulerion graph is Hamiltonian.

(e) The chromatic number of $K_{m,\ n,}$ where $m < n$, is m.

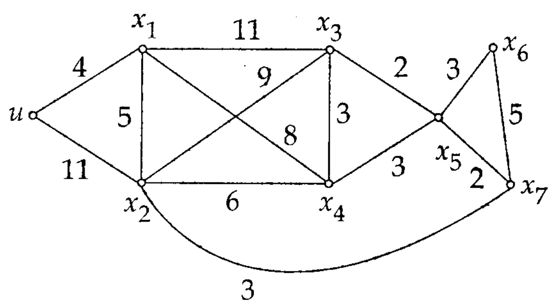
2. (a) Let u and w be distinct vertices in a connected graph. Prove that every u-w walk contains a u-w path. **3**

(b) Draw a diagram of the graph G with vertex set $V(G) = \{1, 2, 3, 4, \ldots\ldots, 10\}$ in which vertices m and n are adjacent if m and n are not relatively prime. **4**

(c) If G is a 2-connected graph, then prove that the graph G' obtained by subdividing an edge of G is also 2-connected. **3**

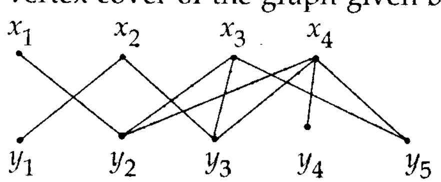
3. (a) The non-negative integers $d_1, d_2, \ldots\ldots, d_n$ are vertex degrees of some graph if and only if Σd_i is even, prove. **4**

(b) In the following weighted graph given below, find the shortest path from the starting vertex u to any other vertex in the graph. **6**

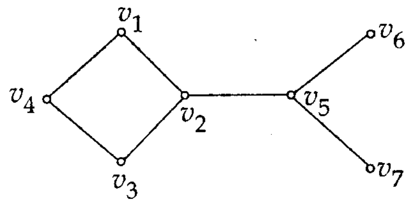


4. (a) Evaluate the connectivity of the complete graph K_n and of the complete bipartite graph $K_{n,\,n}$. **4**

(b) Find maximum matching and minimum vertex cover of the graph given below : **3**



(c) Identify the cut vertices and cut edges of the following graph : 3



Also draw the induced subgraphs obtained by removing

(i) the vertex v_2

(ii) the edge v_1v_2

5. (a) Prove that the center of a tree is either K_1 or K_2. 4

(b) If $\tau(G)$ denotes the number of spanning trees of a graph G and it eEE(G) is not a loop, then prove that 3

$\tau(G) = \tau(G-e) + \tau(G.e)$.

(c) Prove that a plane graph G is bipartite if and only if its dual is Eulerian. 3

6. (a) Let G be a simple graph and u and v are distinct non-adjacent vertices of G with $d(u) + d(v) \geqslant n(G)$. Prove that G is Hamiltonian if and only if $G + uv$ is Hamiltonian. 5

(b) Exhibit a graph G with a vertex v so that 5

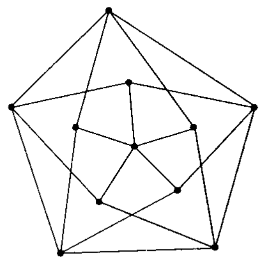
$x(G-v) < x(G)$ and

$x(\overline{G}-v) < x(\overline{G})$

7. (a) Prove that $e(G) \leq 3n(G) - 6$ if G is a simple planar graph. **3**

(b) Prove that, for $k > 0$, every k-regular bipartite graph has a perfect matching. **3**

(c) Consider the following graph : **4**



(i) Find the girth of the graph.

(ii) Write down a 5-cycle in the graph.

(iii) Is the graph isomorphic to the Petersen graph ? Justify your answer.