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MMT-009

M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

Term-End Examination

June, 2013 00922

MMT-009 : MATHEMATICAL MODELLING

Time : 1½ hours *Maximum Marks : 25*
(Weightage : 70%)

Note : *Answer **any five** questions. Use of calculator is **not** allowed.*

1. (a) A model of the population is based on the following assumptions : **3**

(i) There is no exploitation and no restocking.

(ii) The proportionate birth rate is a decreasing linear function of the population.

(iii) The proportionate death rate is an increasing linear function of the population.

When the population was 4000, the proportionate birth rate was 10% and the proportionate death rate was 70%. When the population was 3000, the proportionate birth rate was 30% and the proportionate death rate was 60%.

Show that the model based on these assumptions and the above data follows the logistic model. Find the equilibrium.

(b) Five securities have the following expected returns. 2

A = 12%, B = 18%, C = 16%,
D = 20%, E = 15%

Calculate the expected return for a portfolio consisting of all the five securities where the portfolio weights are 15% each in A and B, 20% each in C and D and 30% in E.

2. (a) The deviation g(t) of a patient's blood glucose concentration from its optimal concentration satisfies the differential equation 3

$$3\,\frac{d^2g}{dt^2} + 8\,\alpha\,\frac{dg}{dt} + 27\,\alpha^2 g = 0 \text{ for } \alpha \text{ being}$$

a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time t is measured in minutes. Identify the type (over damped, under damped or critically damped) of this differential equation. Find the condition on α for which the patient is normal.

(b) A Portfolio P has two securities 1 and 2. If the correlation coefficient $\rho_{12} = -1$, the standard deviation of portfolio P is equal to the sum of weighted average of the standard deviation of its component securities. Classify this statement as true or false giving reasons for your answer. 2

3. The yearly fluctuations in the ground water table is believed to be dependent on the annual rainfall and the volume of water pumped out from the basin. The data collected on these variables for four consecutive years is given below : 5

Water table (in cm)	Annual rainfall (in cm)	Ground water volume Pumped out (in cm^3)
10	3	7
9	4	8
7	5	9
4	7	7

Use the method of least square to find a linear regression equation that best fit the data.

4. Consider the discrete population growth model given by 5

$$x_{n+1} = x_n \exp\left[a \left(1 - \frac{x_n}{K} \right) \right]$$

for a population x_n, where K is the carrying capacity and a is a positive parameter. Determine the non - negative steady states and discuss the stability of the model for $0 < a < 2$. Also find the first bifurcation value of the parameter.

5. Do the stability analysis of the following interacting system of species under the effect of toxicant, when the concentration of the toxicant in the environment is assumed to be constant. 5

$$\frac{dN_1}{dt} = r_1 N_1 - \alpha_1 N_1 N_2 - d_1 C_0 N_1$$

$$\frac{dN_2}{dt} = r_2 N_2 - \alpha_2 N_1 N_2 - d_2 V_0 N_2$$

$$\frac{dC_0}{dt} = k_1 P - g_1 C_0 - M_1 C_0$$

$$\frac{dV_0}{dt} = k_2 P - g_2 V_0 - M_2 V_0$$

Under the initial conditions

$N_1(0) = N_{10}$, $N_2(0) = N_{20}$, $C_0(0) = 0$, $V_0(0) = 0$.

The variables and parameters notation in the above system of equations are

$N_1(t)$, $N_2(t)$ = Density of two different populations

$C_0(t)$ = Concentration of the toxicant in the individual of the population $N_1(t)$.

$V_0(t)$ = Concentration of the toxicant in the individual of the population $N_2(t)$.

P = Concentration of the toxicant in the environment and is constant.

r_1, r_2 are the birth rates; α_1, α_2 are the predation rates; d_1 is the death rate due to C_0 ; d_2 is the death rate due to V_0, k_1, k_2 are uptake; g_1, g_2 are loss rates; m_1, m_2 are depuration rates. Here r_1, r_2 α_1, α_2, d_1, d_2, k_1, k_2, g_1, g_2, m_1, m_2 and P are all positive constants.

6. A petrol station has two pumps. The service time follows the exponential distribution with mean 4 minutes and cars arrive for service in a poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What proportion of time the pump remain idle ? If the customer will wait and serviced in turn, what is the expected waiting time ? 5
