No. of Printed Pages : 5

MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination 00842

June, 2013

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question No. **1** is **compulsory**. Do **any four** questions out of question nos. **2** to **7**. Use of calculator is **not** allowed.*

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1. State whether the following statements are true or false. Justify your answer with the help of a short proof or a counter example. **2x5=10**

(a) The solution of differential equation $xy'' - y' + xy = 0$ is $y = AJ_1(x)$, where $J_1(x)$ is a Bessel function of order one.

(b) The initial-value problem

$$\frac{dy}{dx} = \frac{y+1}{x^2},\ y(0)=1$$ has an infinity of solutions.

(c) Crank-Nicolson method for one-dimensional heat equation is unstable.

(d) The interval of absolute stability for all second order Runge-Kutta methods is $]-2.78, 0[$.

(e) Representation of $f(x) = e^{-kx}$, $x \geq 0$, k a positive constant, by a Fourier sine integral is $f(x) = e^{-kx} = \frac{2}{\pi} \int_0^{\infty} \frac{\alpha \sin \alpha x}{k^2 + \alpha^2} d\alpha$.

2. (a) Using the method of variation of parameters, determine the appropriate Green's function for the boundary - value - problem $y'' + y + f(x) = 0$, $y'(0) = 0$, $y(2) = 0$ and express the solution as a definite integral. 4

(b) Find an approximate value of y (0.6) for initial value proplem $y^1 = x - y^2$, $y(0) = 1$ using multistep method $y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1})$ with step length $h = 0.2$. Compute the starting value using Taylor series second order method with same steplength. 4

(c) Define the triangular shape functions in a finite element method of solving a partial differential equation. 2

3. (a) Using Laplace transforms, solve the p .d.e. 5

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$$ subject to the conditions

$u(0, t) = 0$, $u(5, t) = 0$ $u(x, 0) = \sin(\pi x)$.

(b) Expand $f(x) = x^3 - 3x^2 + 2x$ in a series of the 3

form $\sum_{n=0}^{\infty} a_n H_n(x)$, where $H_n(x)$ is the

Hermite polynomial of degree n in x.

(c) For single - step methods, what is the numerical error at the nodal point ? When is the numerical single-step method said to be stable ? 2

4. (a) Using the five-point formula and assuming 6

the uniform step length $h = \frac{1}{3}$ along the

axes, find the solution of $\nabla^2 u = x^2 + y^2$ in R

where, R is the triangle $0 \le x \le 1$, $0 \le y \le 1$, $0 \le x + y \le 1$. On the boundary of the triangle $u(x, y) = x^2 - y^2$.

(b) If L^{-1} denotes inverse Laplace transform, 4

find $L^{-1}\left[ln\left(1 + \frac{1}{s^2}\right)\right]$.

5. (a) Determine the interval of absolute stability for the method. **5**

$$y_{i+1} = y_{i-1} + \frac{h}{3}(7\, y'_i - 2\, y'_{i-1} + y'_{i-2})$$

when applied to the test equation $y' = \lambda y,\ \lambda < 0$.

(b) Find the solution to the initial boundary value problem, subject to the given initial and boundary conditions, **5**

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\quad u(x,\ 0) = 2x \text{ for } x \in \left[0,\ \frac{1}{2}\right]$$

and $u\ (x,\ 0) = 2\ (-x)$ for $x \in \left[\frac{1}{2}, 1\right]$,

$u(0,\ t) = 0 = u(1,\ t)$.

Using Schmidt method with $\lambda = \frac{1}{6}$ and $h = 0.2$.

6. (a) Bessel's function of the first kind of order $(-n)$ is **3**

$$J_{-n}\ (x) = \left(\frac{x}{2}\right)^{-n} \sum_{m=0}^{\infty} \frac{(-1)^m}{\underline{|m}\ \overline{|}(m-n+1)} \left(\frac{x}{2}\right)^{2m}.$$

Using this definition show that

$$J_{-\frac{1}{2}}\ (x) = \sqrt{\frac{2}{\pi x}} \cos x.$$

(b) Find the power series solution of the differential equation 7

$$x^2 \frac{d^2 y}{dx^2}+\left(x+x^2\right) \frac{dy}{dx}+(x-9)\, y=0$$

about $x=0$.

7. (a) Solve the initial value problem $y' = -2xy^2$, $y(0)=1$ with $h=0.2$ on the interval [0, 0.4] using predictor - correctors method. 4

$$P : y_{i+1}=y_i+\frac{h}{2}\left(3\, y'_i-y'_{i-1}\right)$$

$$C : y_{i+1}=y_i+\frac{h}{2}\left(y'_{i+1}+y'_i\right).$$

Perform two corrector iterations per step and use $y\,(x)=\dfrac{1}{1+x^2}$ to obtain the starting value.

(b) Obtain the general solution of the differential equation 3
$(x+3)^2\, y'' - 4\,(x+3)\, y' + 6y = \ln\,(x+3)$.

(c) Using convolution theorem, find the Fourier inverse of the function $\dfrac{1}{(i\alpha+k)^2}$, $k>0$. 3