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MMT-005

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

June, 2013

MMT-005 : COMPLEX ANALYSIS

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. **1** is **compulsory**. Attempt **any three** questions from question number **2** to **5**. Use of calculator is **not** allowed.*

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1. State giving reasons whether the following statements are **true** or **false** : **5x2=10**

(a) $e^{\bar{z}}$ is nowhere analytic.

(b) $\Sigma \frac{2^n z^n}{n!}$ has radius of convergence ∞.

(c) The harmonic conjugate of $u(x, y) = x^2 + y^2$ does not exist.

(d) $\int_C \frac{\sin z \, dz}{4z + \pi} = \frac{-\sqrt{2}\,\pi i}{4}$ where C is a unit circle.

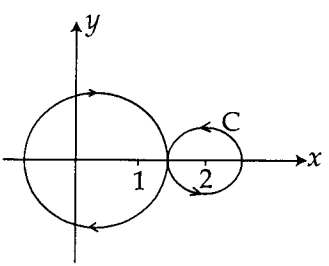
(e) $f(z)=\cos\left(\frac{1}{z-1}\right)^2$ has a pole of order two at $z=1$.

2. (a) Evaluate $\int_C \mathrm{Re}\, z\, dz$ where $C(t)=t+it^2$, from $z=0$ to $z=1+i$. 2

(b) Find the bilinear transformation which takes the points 1, 0, ∞ to -1, i, $-i$. Also find the fixed points of the transformation, if any. 3

3. (a) Evaluate $\int_C \frac{3z+1}{z(z-2)^2}\, dz$ 3

where C is shown in the figure below.



(b) Expand $f(z)=\frac{1}{z\,(z-1)}$ in a Laurent series about $z=1$ valid for $|z-1|>|$. 2

4. (a) Let C denote the circle $|z| = 2$, described in the counter-clockwise direction. Show that 3

$$\left| \int_C \frac{\text{Log } z}{z^2} \, dz \right| \leq \pi (\pi + \ln^2).$$

(b) Find the maximum modulus of $f(z) = 2z + 5i$ on the closed circular region defined by $|z| \leq 2$. 2

5. Evaluate 5

$$\int \frac{(x+1)\cos x}{x^2 + 4x + 5} \, dx.$$