00892

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

June, 2013

MMT-003 : (ALGEBRA)

Time : 2 hours *Maximum Marks : 50*

Weightage 70%

Note : *Question no. 1 is* **compulsory**. *Attempt* **any four** *questions from question no. 2 to question no. 6. Use of Calculator is* **not** *allowed.*

1. State which of the following statements are **true** and which are **false**. Given reasons for your answer : 10

   (a) The eigen values of a matrix $A \epsilon\ SO_3$ are 1 and -1.

   (b) There exists only one injective homomorphism between two free groups.

   (c) There exists an irrational number α such that both α and α^2 satisfy a cubic polynomial with rational coefficients.

   (d) If G is a non-abelian group, then it has no irreducible representations.

   (e) Any cubic irreducible polynomial in $F_7[x]$ must divide $x^7 - x$.

2. (a) Show that a group of order 132 is not simple. 5

(b) For the dihedral group D_6 of order 12, 5

$D_6 = \{x^i y^j | x^6 = y^2 = 1,\ yxy^{-1} = x^{-1}\}$

find the character of the representation for the action of D_6 on $D_6 / \langle x^2 \rangle$ where $\langle x^2 \rangle$ is the cyclic subgroup of D_6 generated by x^2. How many times does the trivial representation occur in this ?

3. (a) Find the orders of all conjugacy classes in S_4. How many of the conjugacy classes have order 6 and contain odd permutations ? 4

(b) Find a grammar that generates 2

$\{b, aba, aabaa, aaabaaa......\}$.

(c) Let G be a finite group V be a finite dimensional vector space over **C**. 4

Let $\rho : G \to GL(V)$ be a group representation and W be a G - invariant subspace of V such that $V = W \oplus W'$ for some subspace W' of V. Will W' always be G-invariant ? Justify your answer.

4. (a) Determine the number of non-isomorphic abelian groups of order 300. 3

(b) Consider the permutation $\sigma = (1357)(2468) \in S_8$. Express σ as a product of three cycles. 4

(c) Check whether $i \in \mathbf{Q}\left(\sqrt{-2}\right)$ or not. 3

5. (a) Solve the following simultaneous system of congruences : 4

$$x \equiv 2(\text{mod } 4)$$

$$x \equiv 1(\text{mod } 5)$$

$$x \equiv 8(\text{mod } 11)$$

(b) Construct an element of order four in the multiplicative group of the finite field of 17 elements. 2

(c) Show that there is a unique irreducible polynomial of degree 2 over $\mathbf{F_2}$. Further, if α is a root of this polynomial, then show that σ is a field automorphism of $\mathbf{F}$ (α), where $\sigma(\alpha) = \alpha + 1$. 4

6. (a) If F is a field and α is transcendental over F, prove that $F(x)$ is isomorphic to F (α) as fields. 5

(b) Decompose M_2 $(\mathbf{C})$ into orbits under the action of $GL_2(\mathbf{C})$. by conjugation. 5
