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ET-101(A)

B.Tech. Civil (Construction Management)/ B.Tech. Civil (Water Resources Engineering) B.Tech. (Aerospace Engineering) BTCLEVI/BIMEVI/BTELVI/BTECVI/BTCSVI

Term-End Examination

June, 2013

00082

ET-101(A) : MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* **compulsory***. Use of scientific calculator is* **permitted***.*

1. Answer *any five* of the following : **5x4=20**

(a) Find $\lim\limits_{x \to 0} \dfrac{x^2}{\sec x - 1}$.

(b) Let $f(x) = ax^2 + bx + c$. If p and q are two real numbers such that $f(p) = f(q)$, prove that $f\left(\dfrac{p+q}{2}\right) = 0$.

(c) Expand $y = \log_e(1 + x)$ in powers of x up to x^5 using Maclaurin's series expansion.

(d) Find the equation of the normal to the parabola $(y-1)^2 = 4a(x-2)$ at the point (x_1, y_1)

(e) Derive necessary condition for a maximum and minimum of a function at a point. Show that the condition is not sufficient. What are sufficient conditions ?

(f) Find the height of a right circular cylinder with greatest lateral surface area that may be inscribed in a given sphere of radius r.

(g) Use differential to find the approximate value of $\sqrt{125}\ \sqrt[4]{15}$.

2. Answer *any four* of the following : **4x4=16**

(a) Find the area between the parabola $y^2 = 4ax$ and the chord $y = mx$.

(b) Find the area of the region outside the circle $r = 2$ and inside the lemniscate $r^2 = 8 \cos 2\theta$

(c) Use Simpson's rule to evaluate $\int_0^{\pi} \frac{\sin x}{x} dx$ by taking $n = 4$.

(d) Find the length of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ between $\theta = 0$ and $\theta = 2\pi$.

(e) Find the volume of the solid generated by revolving the curve $x^{2/3} + y^{2/3} = a^{2/3}$ about x-axis.

(f) Solve $y(x^2y + e^x)dx - e^x dy = 0$.

3. Answer **any four** of the following : **4x4=16**

(a) Show that $\left(\vec{a}\times\vec{b}\right)\times\vec{c} = \vec{a}\left(\vec{b}\times\vec{c}\right)$ if and only if $\vec{a}$ and $\vec{c}$ are collinear vectors.

(b) Find the directional derivative of $f(x, y, z) = 2x^2+3y^2+z^2$ at the point $p(2, 1, 3)$ in the direction of the vector $\vec{a} = \vec{i} - 2\hat{k}$.

(c) Show that the vector field defined by

$$\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$$ is

irrotational. Find a scalar potential u such that $\vec{F} = \text{grad}\,u$.

(d) Find the work done in moving a partical once round the circle $x^2+y^2=9$ in the XOY plane when the field of force is given by

$$\vec{F} = (2x-y-z)\hat{i} + (x+y-z^2)\hat{j} + (3x-2y+42)\hat{k}.$$

(e) State Green's Theorem in a plane. Use it to evaluate $\oint[(x^2-2xy)dx+(x^2y+3)dy]$

around the boundary C of the region $y^2=8x$, $x=2$.

(f) For any closed surface S, prove that

$$\iint_S \left[x(y-z)\hat{i} + y(z-x)\hat{j} + 2(x-y)\hat{k} \right] d\vec{s} = 0.$$

4. Answer **any six** of the following : **6x3=18**

(a) Determine an orthonormal basis of the subspace
$S\{(1,1,0,1), (-1,1,1,-1), (0,2,1,0), (1,0,0,0)\}$.

(b) Let A and B be square matrices such that $AB + A + I = 0$. Prove that A is non-singular and $A^{-1} = -I - B$.

(c) Using the properties of determinants evaluate

$$|A| = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 3 & 3 & 3 \\ 2 & 3 & 4 & -3 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

What is the rank of [A].

(d) What is matrix eigen-value problem ? State any three properties of the eigen values and prove them.

(e) Define the rank of a matrix. Find the rank of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

(f) Find the eigen values and the eigen vector of the matrices

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(g) Check for the consistency of the following system of linear equations :

$2x - y + z = 4$

$x + 2y - 3z = 8$

$4x + 3y - 5z = 10$

(h) Reduce $x^2 + 3y^2 + 3z^2 - 2yz$ to its canonical form.

(i) State Cayley-Hamilton Theorem and verify it for the matrix $\begin{bmatrix} 1 & 5 \\ 2 & 9 \end{bmatrix}$.
