

M.Sc. ACTUARIAL SCIENCE
(MSCAS)

Term-End Examination

June, 2012

MIA-001 (F2F) : FINANCIAL MATHEMATICS

Time : 3 hours

Maximum Marks : 100

*Note : In addition to this paper you should have available the
Formulae and Table and your own electronic calculator.*

SECTION - A

(Answer *any five* questions)

1. (a) Given $d = 7\%$ p.a, find 4

(i) $a_{\overline{3}|}^{..(12)}$

(ii) $(\bar{I}a)_{\overline{10}|}$

(b) Define the following terms : 4

(i) Simple interest

(ii) Compound interest

(iii) Nominal rate of interest

(iv) Force of interest

2. (a) A loan of Rs. 10,000 is repaid over a five - year period by level monthly repayments in arrears of Rs. 250. Calculate: 4
- (i) the flat rate of interest per annum.
- (ii) the APR on the transaction.
- (b) Prove that 4

$$S_{\overline{n}|}^{(p)} = \frac{(1+i)^n - 1}{i} \cdot (p)$$

3. At time $t=0$ an investor purchased an annuity - certain which paid her Rs.10,000 p.a annually in arrear for three years. The purchase price paid by the investor was Rs. 25,000. 8

The value of the retail price index at various times was as shown in the table below :

Time t (years) :	t = 0	t = 1	t = 2	t = 3
Retail price index :	170.7	183.3	191.0	200.9

Calculate to the nearest 0.1%, the following effective rates of return per annum achieved by the investor from her investment in the annuity.

- (a) the real rate of return.
- (b) the money rate of return.

4. An university student receives a 3 - year 8
sponsorship grant. The payments under the grant
are as follows :

Year 1, Rs. 5000 per annum paid continuously

Year 2, Rs. 5000 per annum paid monthly in
advance

Year 3, Rs. 5000 per annum paid half yearly in
advance.

Calculate the total present value of these payments
at the beginning of the first year using a rate of
interest of 8% per annum convertible quarterly.

5. The table below shows the progress of a lottery 8
winners investment portfolio for the 2006 calendar
year.

	Fund value	Cashflow
1 January	Rs. 75,000	-
31 March	Rs. 90,000	-
1 April	Won Lottery, Winning Invested	Rs.27,00,000
30 September	Rs. 26,00,000	-
1 October	Funds withdrawn to pay off gambling debt	Rs.25,00,000
31 December	Rs. 1,25,000	-

Calculate (to the nearest %) the MWRR and the
TWRR for 2006 for this portfolio and comment on
your answers.

6. An investor purchased a 5 - month forward contract on 1 January 2004 to buy 1000 shares at the end of the contract. The price of a share on 1 January 2004 is Rs. 50. Dividends are received continuously and the dividend yield is 6% p.a.

The risk free rate of interest is 4% p.a. effective and there is no arbitrage.

- (a) Calculate the forward price. 4
- (b) The investor decides to sell the contract on 1 April 2004 when the price of the stock is Rs. 49.50 per share. Calculate the value of the contract at this time. 4

7. Define the following terms : 8

- (a) Future.
- (b) Interest rate swap.
- (c) Currency swap.
- (d) Option.

SECTION - B

(Answer *any four* questions)

8. 5 years ago , a borrower took a home loan of Rs. 600000 from a bank at rate of interest of 10% per annum effective. Loan is repayable by monthly instalments of Rs. X each paid in arrears over 15 - year period.
- (a) Calculate X 3
- (b) Calculate the interest and capital components of 19th instalments. 4
- (c) The rates of interest on home loan have fallen during past 5 years, In order to retain the customer, the bank makes an offer to the borrower to make a new loan equal to the outstanding loan at the end of 5 years at rate of interest of 8% per annum effective. The bank charges a processing fee equal to 4% of the original loan. The first loan is repaid and new loan is repayable over remaining 10 years period by monthly instalments in arrear.
- (i) Calculate the revised monthly instalment of the new loan. 5

- (ii) Calculate the present value of reduction in monthly instalment at the rate of interest of 8 % per annum effective and show that the offer is profitable to borrower.

9. A piece of land is available for sale for Rs. 50,00,000. A property developer, who can lend and borrow money at a rate of 15% per annum, believes that she can build housing on the land and sell it for a profit. The total cost of development would be Rs. 70,00,000 which would be incurred continuously over the first two years after purchase of the land. The development would then be complete.

The developer has three possible project strategies. She believes that she can sell the completed housing :

In three years time for Rs. 1,65,00,000

In four years time for Rs. 1,80,00,000

In five years time for Rs. 2,05,00,000

The developer also believes that she can obtain a rental income from the housing between the time that the development is completed and the time of sale. The rental income is payable quarterly in advance and is expected to be Rs. 5,00,000, in the first year of payment. Thereafter, the rental

income is expected to increase by Rs. 50000 per annum at beginning of each year that the income is paid.

- (a) Determine the optimum strategy if this is based upon using net present value as the decision criterion. 9
- (b) If the housing is sold in six years time, the developer believes that she can obtain an internal rate of return on the project of 17.5% per annum. Calculate the sale price that the developer believes that she can receive. 6

10. One year ago, a loan was issued bearing interest at the rate of 14% p.a. payable half yearly in arrears. The loan will be redeemed at Rs. 110% in nine years time.

- (a) The loan was issued at a price such that an investor subject to income tax at 35%, but not subject to capital gain tax, would obtain a net yield of 9.5% p.a. Calculate the issue price for Rs. 100 nominal. Interest rates has now fallen. The investor has decided to sell and has found a potential buyer, who is subject to income tax at 25% and capital gain tax at 35%, and who is prepared to buy the stock provided that he obtain a net yield of atleast 10% p.a. 4

- (b) What is the best price (per Rs. 100 nominal) the original investor can expect to obtain from the potential buyer ? 5
- (c) Calculate the net running yield obtained by the buyer. 2
- (d) Calculate the net yield that will be obtained by the original investor if the loan is sold to the buyer at the price determined in (b). 4
11. (a) Explain what is meant by market segmentation and expectation theory according to the term structure of interest rate. 4
- (b) The n - year spot rate of interest, i_n , is given by
- $$i_n = a - bn$$
- for $n=1,2$ and 3 , and where a and b are constants.
- The one year forward rates applicable at time 0 and at time 1 are 6.1% per annum effective and 6.5% per annum effective respectively. The 4 - year par yield is 7% per annum.

Stating any assumptions :

- (i) Calculate the values of a and b 5
- (ii) Calculate the price per Rs. 1 nominal at time 0 of a bond which pays annual coupons of 5% in arrear and is redeemed at 103% after 4 years. 6
12. (a) A company has to pay Rs. 2000 $(10 - t)$ at the end of year t , for $t = 5, 6, 7, 8, 9$. It values these liabilities assuming that there will be a constant effective annual rate of interest of 6% p.a. 4
- (i) Express the present value of the liabilities in terms of level and increasing annuities.
- (ii) Hence calculate the present value of the liabilities.
- (b) The company wants to immunise its exposure to the liabilities by investing in 2 bonds : 11
- Bond A pays coupons of 5% p.a. annually in arrears and redeemable at par in 15 years time.
- Bond B is a zero coupon bond that is redeemable at par in 5 years time.

The gross redemption yield on both stocks is the same as the interest rate used to value the liabilities.

- (i) Calculate the amount that the company should invest in each of the two bonds to ensure that the present value and volatility of the assets are equal to those of the liabilities.
- (ii) What other condition is required for immunisation ?

13. In any year, the rate of interest on funds invested with a particular company has mean value j and standard deviation S , and is independent of the rates of interest in all previous years.

- (a) Derive formulae for the mean and variance of the accumulated value after n years of a single investment of 1 at time 0. 5
- (b) Let it be the rate of interest earned in the t^{th} year. Each year the value of $(1 + i_t)$ is lognormally distributed, with parameter $\mu = 0.04$ and $\sigma^2 = 0.09$.

- (i) Show that n , the number of years that must elapse before the accumulation of a lumpsum invested at time 0 has a 75% probability of at least doubling is size, satisfies :

$$0.04n - 0.2024\sqrt{n} - \ln 2 = 0$$

- (ii) Hence, or otherwise, calculate the value of n . **10**