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**MMT-009**

# M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

## Term-End Examination

00923

**June, 2012**

## MMT-009 : MATHEMATICAL MODELLING

*Time : 1½ hours* | *Maximum Marks : 25*

*Note : Do **any five** questions. Use of calculator is **not** allowed.*

1. (a) Formulate the model for which the reproductive function of the cancer cells in the tumour surface is given by $\phi(C) = \frac{C-1}{1-2C}$, $C \neq \frac{1}{2}$ together with initial conditions $C = 20 \times 10^5$ at $t = 0$. Also find the density of the cancer cells in the tumour's surface area at $t = 45$ days. **3**

   (b) Given a set of seven securities with portfolio values : **2**

   $$wi's = \frac{1}{3}, \frac{2}{3}, \frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}.$$

   Find a feasible set of portfolio of these securities.

2. (a) A colony of bacteria increases at a rate that is proportional to the number of bacteria in the colony. If the population quadraples in two years, find the size of the colony after eight years. 2

(b) Following is the data for number of years students studied a subject and the score he/she received in that subject. 3

| Number of years | Test score |
|---|---|
| 3 | 57 |
| 4 | 78 |
| 4 | 72 |
| 2 | 58 |
| 5 | 89 |
| 3 | 63 |
| 4 | 73 |
| 5 | 84 |
| 3 | 75 |
| 2 | 48 |

Fit the least square line to this data. What is the score of the student who has studied for two years, according to this line ?

3. Consider the following prey and predator interacting system under the effect of toxicant, where the concentration of the toxicant in the environment is assumed to be constant. 5

$$\frac{dN_1}{dt} = r_0 N_1 - r_1 C_0 N_1 - bN_1 N_2$$

$$\frac{dN_2}{dt} = -d_0 N_2 + \beta(C_0) bN_1N_2$$

$$\frac{dC_0}{dt} = k_1P - g_1C_0 - m_1C_0$$

along with initial conditions: $N_1(0) = N_{10}$, $N_2(0) = N_{20}$, $C_0(0) = 0$ and $\beta(C_0) = \beta_0 - \beta_1 C_0$ where $\beta(C_0)$ is the conversion coefficient depending upon $C_0$. The variables and parameters notations in the above system of equations are :

$N_1(t)$ = Density of prey population.

$N_2(t)$ = Density of predator population.

$C_0(t)$ = Concentration of toxicant in the individuals of the population.

$P$ = Concentration of toxicant in the environment and is constant.

$r_0$ is the birth rate, $r_1$ is the death rate due to $C_0$, b is the predation rate, $d_0$ is the death rate, $k_1$ is the uptake rate, $g_1$ is the loss rate of the toxicant and $m_1$ is the depuration rate. Here $r_0$, $r_1$, b, $d_0$, $k_1$, $g_1$ and $m_1$ are all positive constants .

Reformulate the above system under the assumption that the predators are not affected by the toxicant because they are generally strong and then do the stability analysis of the reformulated system.

4. (a) Classify the following into linear and non-linear models. Justify your classification. 2

(i) Logistic growth model.

(ii) One-dimensional diffusion equation where the diffusion coefficient depends on the concentration of diffusing substance.

(b) A model for insect population leads to the difference equation 3

$$N_{R+1} = \frac{\lambda N_R}{1 + aN_R}$$

where $\lambda$ and a are positive constants.

(i) Write the equation in the form $N_{R+1} = N_R + R(N_R)N_R$ and hence identify the growth rate.

(ii) Plot the graph of $R(N_R)$ as a function of $N_R$.

(iii) Express the intrinsic growth rate, r and the carrying capacity K, for this model, in terms of the parameters a and $\lambda$.

5. Ships arrive at a port at the rate of one in every 4 hours with exponential distribution of inter arrival times. The time a ship occupies a berth for unloading has exponential distribution with an average of 10 hours. If the average delay of ships waiting for berths is to be kept below 14 hours, how many berths should be provided at the port ? 5

6. (a) How are the returns on the two securities A and B related when **2**

(i) the covariance between A and B is positive ?

(ii) there is zero correlation between A and B ?

(b) Let G(t) be the amount of the glucose in the blood stream of a patient at time t. The glucose is infused into the blood stream at a constant rate of R gm/min. At the same time the glucose is converted and removed from the blood stream at a rate proportional to the amount of glucose present. If the initial concentration of the glucose in the blood stream was $G_0$ then find the concentration at any time t. Also find the limiting value of the concentration. **3**

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