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MMT-005

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

June, 2012

MMT-005 : COMPLEX ANALYSIS

Time : 1½ hours *Maximum Marks : 25*

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*Note : Question **1** is **compulsory**. Attempt **any three** questions from question 2 to 5. Use of calculator is **not** allowed.*

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1. State giving reasons whether the following statements are *true* or *false* : **5x2=10**

   (a) The image of the disc $|z-i| \leq 1$ under the linear fractional transformation $T(z)=\frac{i}{z}$ is contained in $|w| \geqslant \frac{1}{2}$.

   (b) $f(z)=z^2+\bar{z}$ is nowhere analytic.

   (c) $f(z)=\sin hz \cos hz$ is bounded function.

   (d) The function $4(x, y)=e^{4x} \cos 2y$ is the real part of an analytic function.

(e) $\oint_C \frac{1}{z^2 - 3z + 2} dz = \frac{1}{2}$, where C is the circle $|z| = \frac{1}{2}$.

2. (a) Derive the formula 3

$$\frac{1}{2\pi}\int_0^{2\pi} \cos^{2n} t \, dt = \frac{1.3.5........(2n-1)}{2.4.6.......(2n)}$$

by integrating the function $\frac{1}{z}\left(z + \frac{1}{z}\right)^{2n}$ around the unit circle $C : z = e^{it} (0 \le t \le 2\pi)$.

(b) Let $f(z)$ be analytic in a domain D. Prove that $f(z)$ is constant if imaginary part of $f(z)$ is constant. 2

3. (a) Find the upper bound for $\left|\oint_C f(z) dz\right|$ where $|f(z)| \le 2$ on the circle $|z| = 3$. 1

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{\cos\theta + 2\sin\theta + 3}$. 4

4. (a) Expand $f(z) = \frac{z^2 - 2z + 2}{(z - 2)}$ in a Laurent series valid for the annular domain $|z-1|>1$. 3

(b) Evaluate $\oint_C \frac{2z + 6}{z^2 + 4} dz$ where C is the circle $|z-1| = 2$. 2

5. (a) Consider $f(z) = z(z-2)$ and the closed circular region $R = \{z : |z| \leq 2\}$. Find points in R where $|f(z)|$ has its maximum and minimum values. 3

(b) Find the image of the triangle with vertices at $-\frac{1}{\sqrt{2}} + \frac{i\sqrt{3}}{2}$, $-\sqrt{2}$ and 0 under the transformation $w = e^{\frac{i\pi}{4}} (z + \sqrt{2})$. 2