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MMT-004

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

June, 2012

MMT-004 : REAL ANALYSIS

Time : 2 hours *Maximum Marks : 50*

Note : *Question no. 1 is **compulsory**. Do **any four** questions out of questions no. 2 to 7.*

1. State, whether the following statements are **TRUE** or **FALSE**. Give reasons for your answer. **5x2=10**

   (a) In a metric space every Cauchy sequence is convergent.

   (b) The function $f : R \to R^2$ defined by.

   $f(x) = (x, x \sin \frac{1}{x})$ is not differentiable at $x = 0$.

   (c) The system $R : f \to g$ given by :

   $g(t) = \int_{\infty}^{t} f(\tau)\, e^{-(t-\tau)} d\tau$ is stable.

(d) The set $(0, \infty)$ is a bounded set in the metric space (R, ρ) where :

$\rho(x, y) = \min\{1, |x-y|\}$

(e) Every Lebesgue integrable function is Riemann integrable.

2. (a) Let X and Y be metric spaces. Show that a map $f: X \to Y$ is continuous if and only if for every closed set $V \subset Y$, $f^{-1}(V)$ is closed in X. 5

(b) Find the derivative of the function f defined by : 3

$f(x_1, x_2) = \left(x_1^2 - x_2, x_1 + 2x_2\right)$ at the point (2, 3).

(c) Which of the following sets are closed in R^2 w.r.t. the standard metric ? 2

(i) $A = \{(x, y) \in R^2 : xy = 0\}$

(ii) $B = \{x, y) \in R^2 : x^2 + y^2 < 1\}$

3. (a) State Faton's lemma. Using this lemma, prove the following result : 5

Let E be a measurable set and let $\{f_n\}$ be an increasing sequence of measurable functions. Let f be defined by :

$f(x) = \lim_{n \to \infty} f_n(x) \ \forall \ x \in E$. Then show

that : $\lim_{n \to \infty} \int_E f_n \, dm = \int_E f \, dm$

(b) Define Fourier transform of a function f, where $f \in L'(R)$. Show that, if $f \in L'(R)$, then its Fourier Transform $\hat{f}$ is continuous on R. 5

4. (a) Let $h(t) = e^{-2t}$. Find the system response to the input function $f(t) = \sum_{k=-2}^{2} \left(\frac{1}{2}\right)^k e^{i5kt}$. 3

(b) Show that the finite sets are the only sets which are compact in a discrete metric space. 2

(c) If X is a connected metric space then, show that any continuous function f on X to the discrete metric space $\{1, -1\}$ is a constant function. Is the converse true ? Justify your answer. 5

5. (a) Verify implicit function theorem for the function : $f(x, y, z) = x + y + z - \sin xyz$ at (0,0,0). 3

(b) Give an example of a vector valued function $f : R^4 \to R^3$ for which the second order derivative exists. 2

(c) Show that for all a, b $\in R$, $m^*\{a, b\} = b - a$. 5

6. (a) Show that a metric space is complete if and only every cauchy sequence in it has a convergent subsequence. 4

(b) Find the directional derivative of the function $f : R^4 \to R^3$ defined by : 6

$f(x, y, z, w) = (x^2y, xyz, x^2 + y^2 + zw^3)$ at $a = (1, -2, 2)$ in the direction of $V = (1, 0, -1, 2)$.

7. (a) Compute the Fourier Series of the function f given by $f(t) = t^2, -\pi \leq t \leq \pi$. 5

(b) Obtain the second order Taylor Series expansion for the function : 5

$f(x,y) = x + 2y + xy - x^2 - y^2$ at the point

$$a = \left(\frac{4}{3}, \frac{5}{3}\right).$$