No. of Printed Pages : 3 **MMT-003**

00791

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

June, 2012

MMT-003 : (ALGEBRA)

Time : 2 hours *Maximum Marks : 50*

*Note : Question no. 1 is **compulsory**. Answer **any four** questions from question no. 2 to question no. 6. Use of calculator is not allowed.*

---

1. State, with reasons, which of the following statements are *true* and which are *false*. **10**

(a) The dihedral group D_6 has exactly six sub groups of order 2.

(b) If F is a free group on two generators x and y, and $F_1 \subset F$ is a subgroup that does not contain x or y. Then F_1 is not isomorphic to F.

(c) There is no transitive action of the group Z_5 on Z_6.

(d) For any natural number n, $n > 1$, $(n-1)! \equiv -1 \pmod{n}$.

(e) There exists a group of order 12 with exactly two irreducible representations of degree 1.

2. (a) Show that a group of order 77 is cyclic. 5

(b) Let $\mathbf{F} = \mathbf{Z}_{13}$ and $\alpha = \sqrt[3]{\overline{10}}$ where $\overline{10} \in \mathbf{Z}_{13}$. 3

Find the minimal polynomial satisfied by α.

(c) Find a solution to the equation $6x \equiv 7 \bmod 11$. 2

3. (a) Let F be any field. Check that the group Gln (F) acts on Mn (F) by conjugation. Is the action transitive ? Justify your answer. 3

(b) Let $F = \dfrac{Z_5[x]}{\langle x^3 + 3x + 2 \rangle}$ and 2

$\alpha = x + \langle x^3 + 3x + 2 \rangle \in F$. Find α^5.

(c) Solve the simultaneous congruence. 5

$x \equiv 3 \bmod 7$

$x \equiv 5 \bmod 8$

$x \equiv 3 \bmod 5$

4. (a) Let $F = \mathbf{F}_5$. Find the order of the conjugancy class of $\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$ in $GL_2(\mathbb{F}_5)$. 3

(b) Let $G = Z_5 \times Z_6 \times Z_9 \times Z_{12}$. Find the elementary divisors and invariant factors of G. 5

(c) Let P be a representation of a group G. Is $g \rightarrow p(gt)$ is a representation of G. Justify your answer. 2

5. (a) Let $D_3 = \{ x, y \mid x^2=1, \; y^3=1, xyx = y^2 \}$. **5**

consider the map $\rho : D_3 \to GL_2(\mathbb{C})$

defined by $\rho(y) = \begin{pmatrix} \cos 2\pi/3 & -\sin 2\pi/3 \\ \sin 2\pi/3 & \cos 2\pi/3 \end{pmatrix}$

$\rho(x) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(i) Check that ρ is a representation.

(ii) Find the character of ρ.

(b) Prove that the subgroup SO_2 of SU_2 is conjugate to the subgroup of diagonal matrices. **3**

(c) Show that $\sqrt{3} \notin Q\left[\sqrt{6}\right]$. **2**

6. (a) Check if 81-7808-902-5 is a valid ISBN number. **3**

(b) Let F be a field and $\alpha \in F$ be such that $[F(\alpha) : F] = 5$. Show that $F(\alpha) = F(\alpha^3)$. **4**

(c) Find the language $L(\Gamma)$ generated the grammar **3**

$\Gamma = (\{a\}, \{g_o\}, \{g_o \to a, \; g_o \to ag_o\}, g_o)$