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MMT-002

00261

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

June, 2012

MMT-002 : LINEAR ALGEBRA

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. 5 is **compulsory**. Answer **any three** questions from question No. 1 to 4. Use of calculators is **not allowed**.*

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1. (a) Let T : $R^4 - R^3$ be a linear transformation given by : **2**

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 + x_3 + 2x_4 \\ x_1 - 2x_2 + x_3 - x_4 \\ 2x_1 - 5x_2 + x_4 \end{bmatrix}.$$

Find the matrix of T with respect to the standard ordered basis of TR^4 and the basis

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\} \text{ of } TR^3.$$

(b) Find an unitary matrix U such that U*AU is upper triangular, where 3

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$

2. (a) Give an example, with justification, of two non-similar 3×3 matrices having the same determinant and trace. 1

(b) How many Jordan canonical forms can there be for a linear operator whose characteristic polynomial is $(x-3)^3(x-5)^2$, and why ? 2

(c) Give an example, with justification, of a normal matrix which is neither Hermitian, nor skew-Hermitian, nor unitary. 2

3. (a) Consider the predator-prey matrix of two populations given by $\begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$. Check whether both populations perish with time or not. 2

(b) Write the spectral decomposition as a linear combination of the projection matrices of the self adjoint operator on C^3 whose matrix representation with respect to the standard basis is given by : 3

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

4. (a) Find the SVD for the matrix **4**

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

(b) Obtain det (e^A), where A is the matrix given in Question No. 3(b). **1**

5. Which of the following statements are true ? Give reasons for your answers : **10**

(a) If A is an $n \times n$ matrix with determinant 1, then A is similar to I_n.

(b) There is a unique $n \times n$ matrix with spectral radius n.

(c) (1, 0, 0), (1, −1, 5), (2, 3, 1), (5, −1, 3) form a Jordan chain of some $T \in L(R^3)$.

(d) If all the eigenvalues of a square matrix are real, then the matrix is self - adjoint.

(e) Given any $m \times n$ matrix A, the eigenvalues of A^*A are the same as those of AA^*.