

03489

**BACHELOR OF TECHNOLOGY IN
MECHANICAL ENGINEERING
(COMPUTER INTEGRATED
MANUFACTURING)**

Term-End Examination

June, 2012

BME-015 : ENGINEERING MATHEMATICS-II

Time : 3 hours

Maximum Marks : 100

Note : Answer any ten questions. All the questions carry equal marks. Use of calculator is permitted.

1. Test the convergence or divergence of the series

$$\frac{\sqrt{2-1}}{3^3-1} + \frac{\sqrt{3-1}}{4^3-1} + \frac{\sqrt{4-1}}{5^3-1} + \frac{\sqrt{5-1}}{6^3-1} + \dots$$

2. Find the general solution of $y'' + xy' - x^2y = 0$ in the form of power series in powers of x .

3. Find the radius of convergence of the series

$$\frac{1}{2}x + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \dots$$

4. Show that for $-\pi < x < \pi$

$$\sin ax = \frac{2 \sin ax}{\pi} \left(\frac{\sin x}{1^2-a^2} - \frac{2 \sin 2x}{2^2-a^2} + \frac{3 \sin 3x}{3^2-a^2} - \dots \right)$$

5. Find a series of cosines of multiple of x which will represent $x \sin x$ in the interval $(0, \pi)$.

6. If n is positive integer, prove that

$$(1 + i\sqrt{3})^n - (1 - i\sqrt{3})^n = 2^{n+1} \cos^{n\pi/3} \quad \text{and}$$

Hence find the value when $n=9$.

7. If $w = \phi + i\psi$ represents the complex potential for

an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$

determine the function ϕ .

8. Obtain the Laurent's series expansion of

$$\frac{e^z}{z(z^2+1)}, \quad 0 < |z| < 1.$$

9. Evaluate the following integrals by contour

integration $\int_0^{2\pi} \frac{d\theta}{(a+b \cos\theta)^2} \quad a > b > 0$

10. Prove that :

$$\int_0^{\pi} \frac{a \, d\theta}{a^2 \sin^2 \theta} = \frac{\pi}{\sqrt{1+a^2}}, \quad a > 0$$

11. Evaluate :

$$\int_0^{\alpha} \frac{x^{a-1}}{x^2+x+1} dx, 0 < a < 2$$

12. Find a transformation which will map a 60° sector of the unit circle in z -plane into the upper half of the w -plane.

13. Find the general solution of $9y'' - 6y' + y = 0$

14. Use the method of Laplace transforms to find the solution of

$$y'' + ay = 6 \cos 3t,$$

$$y(0) = 2, y'(0) = 0$$

15. Find the solution of the heat conduction problem

$$U_{xx} = 4U_t \quad 0 < x < 2 \quad t > 0$$

$$U(0, t) = 0 = U(2, t) \quad t \geq 0$$

$$U(x, 0) = 2 \sin \frac{\pi x}{2} - \sin \pi x + 4 \sin 2\pi x$$
