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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) B.Tech. (Aerospace Engineering)

00289

Term-End Examination

June, 2012

ET-102 : MATHEMATICS III

Time : 3 hours *Maximum Marks : 70*

Note : *Question No.* **1** *is* ***compulsory****. Attempt any other* ***eight*** *questions from Q. No.* **2** *to Q. No.* **15**. *Use of calculator is allowed.*

1. Complete the following : **7x2=14**

(a) The sequence $<x_n>$, where $x_n = \left(1+\frac{1}{n}\right)^{n+1}$,

is monotonically __________ sequence and is bounded and converges to a finite positive limit whose value is __________.

(b) The radius of convergence of the series

$\frac{1}{2}x + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^2 + \ldots\ldots$ is ______.

(c) If $u = 2x - 3x^3 + 9xy^2$ is a harmonic function, then its conjugate function v is __________.

(d) The essential singularity of the function $f(z) = z^n e^{1/z} (1+z)^{-1}$, $n \in N$, is ________ and the residue of $f(z)$ there at is ________.

(e) The value of b for which the differential equation $(yx^{2xy} + x)\, dx + bxe^{2xy}\, dy = 0$ is ________.

(f) The initial boundary value problem $u_{tt} - c^2 u_{xx} = 0$, $0 < x < e$, $t > 0$ with $u(0, t) = 0 = u(e, t)$, $t \geq 0$, $u(x, 0) = f(x)$, $0 \leq x \leq e$, $u_t(x, 0) = g(x)$, $0 \leq x \leq e$ has the solution

$$4(x, t) = \sum_{n=1}^{\infty} \left(A_n \sin \frac{n\pi ct}{e} + B_n \cos \frac{n\pi ct}{e} \right)$$

$\sin \frac{n\pi x}{e}$, where $A_n =$ ________ and $B_n =$ ________.

(g) If l^{-1} denotes Laplace Inverse, then $l^{-1} \left[\frac{s}{s^2 + 6s + 11} \right] =$ ________.

2. (a) Apply Picard's method to find first two approximations to the solution of IVP 3½

$$\frac{dy}{dx} = e^x + e^y \text{ with } y(0) = 0$$

(b) Solve $dx + x\, dy = e^{-y} . \sec^2 y\, dy$. 3½

3. Using Method of Feroberius, find series solution near $x=0$ of the differential equation 7

$$9x\,(1-x)\,\frac{d^2y}{dx^2}-12\,\frac{dy}{dx}+4y=0.$$

4. Using Laplace Transforms, solve $(D^3+3D^2+3D+1)\;y=x^2e^{-x}$ with $y(0)=1$, $y'(0)=0$, $y''(0)=-2$ 7

5. (a) Show that $l\,(t^n)=\frac{\sqrt{n+1}}{s^{n+1}}$, $n>-1$, $s>0$ 3½

(b) Using convolution theorem, evaluate 3½

$$l^{-1}\left\{\frac{1}{(s+1)\,\left(s^2+1\right)}\right\}.$$

6. Find the equation of integral surface of the p.d.e. $(x^2-y^2-z^2)p+2xyq-2xz=0$. 7

7. Using Cauchy's general principle of convergence, show that the sequence $<a_n>$, defined as 7

$a_n=1+\frac{1}{2}+\frac{1}{3}+\_\_\_\_+\frac{1}{n}$ does not converge.

8. (a) Show that the series $1+x+\frac{x^2}{\lfloor 2}+$ —— 3½

converges absolutely for all values of x.

(b) Test the series $\sum \frac{\sin(x+nx)}{n(n+1)}$ for uniform 3½

convergence.

9. Find the half-range cosine series for the function $f(x)=(2x-1)$ for $0<x<1$. 7

10. Find the Fourier series to represent odd function $f(x)=x$ from $-\pi$ to π. 7

11. (a) Find the characteristic function, transfer function and frequency response function of the equation $(D^3+1)\,x=f(t)$. 3

(b) Test the differential equation, $(D^2+2D+7)\,x=f(t)$, for stability. 4

12. Using Hurwitz-Routh Criterion for stability, determine the value of 'a' so that the differential equation whose characteristic function is given by $s^4+as^3+6s^2+4s+1=0$ is stable. 7

13. Find the deflection $u(x, t)$ of the vibrating string of length π, ends fixed and $c^2=1$ with zero initial velocity and $k \sin 2x$ as initial deflection (use the method of separation of variables). 7

14. (a) Find the value of $\int_{C:|z|=1} e^z z^{-2} \, dz$ **3**

(b) Find Taylor series of e^z about $z=1$. **4**

15. Using the method of complex integration, **7**

evaluate $\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta} \, d\theta$.
