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BICE-007

(BTCSVI / BTECVI / BTELVI) B.Tech. (Degree)

Term-End Examination

June, 2012

BICE-007 : MATHEMATICS-III

01425

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**.*
*Use of calculator is **permitted**.*

1. Answer ***any two*** of the following : **2x7=14**

(a) Show that the polar form of Cauchy - Riemann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}\ ;\ \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

Deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.

(b) Prove that $\int_C \frac{1}{z-a}\,dz = 2\pi i$

where C is the circle $|z-a| = r$.

(c) Determine the poles of the function

$$f(z) = \frac{z^2}{(z-1)^2(z+2)}$$

and the residue at each pole.

2. Answer *any two* of the following : **2x7=14**

(a) Calculate the first four moments of the following distribution about the mean.

x :	0	1	2	3	4	5	6	7	8
f :	1	8	28	56	70	56	28	8	1

Also evaluate β_1 and β_2.

(b) Find the correlation co-efficient between x and y for the given values. Find also the two regression lines.

x :	1	2	3	4	5	6	7	8	9	10
y :	10	12	16	28	25	36	41	49	40	50

(c) The two regression equations of the variables x and y are :

$$x = 19.13 - 0.87\ y, \text{ and}$$

$$y = 11.64 - 0.50\ x$$

Find :

(i) mean of x's

(ii) mean of y's, and

(iii) the correlation co-efficient between x and y.

3. Answer *any two* of the following : **2x7=14**

(a) Out of 800 families with 5 children each, how many would you expect to have :

(i) 3 boys (ii) 5 girls

(iii) either 2 or 3 boys ?

Assume equal probabilities for boys and girls.

(b) In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing :

(i) no defective,

(ii) one defective, and

(iii) two defective blades respectively in a consignment of 10,000 packets.

(c) A random sample of 10 boys had the following I.Q :

70, 120, 110, 101, 88, 83, 95, 98, 107, 100

Do these data support the assumption of a population mean I.Q of 100 (at 5% level of significance) ?

4. Answer *any two* of the following : **2x7=14**

(a) Find a root of the equation $x^3 - 4x - 9 = 0$, using the bisection method in four stages.

(b) Using Regula-falsi method, compute the real root of the following equation $x\, e^x - 2 = 0$ correct to three decimal places.

(c) Find by Newton-Raphson's method, a root of the following equation $x^3 - 2x - 5 = 0$, correct to 3 decimal places.

5. Answer *any two* of the following : **2x7=14**

(a) Solve, by Jacobi's iteration method, the equations :

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(b) Evaluate $\int_0^6 \frac{1}{1+x^2}\, dx$ by using Simpson's $\frac{1}{3}$ rule. (Take h = 1).

(c) Using Runge - Kutta method of order 4, find y (0.2) given that $\frac{dy}{dx} = 3x + \frac{y}{2}$, y (0) = 1, taking h = 0.1