M. Sc. (MATHEMATICS WITH

## APPLICATIONS IN COMPUTER

SCIENCE) [M. Sc. (MACS)]

## Term-End Examination

## December, 2022

## MMT-009 : MATHEMATICAL MODELLING

Time : $1 \frac{1}{2}$ Hours<br>Maximum Marks : 25

Weightage : 70\%

Note: (i) Attempt any five questions.
(ii) Use of scientific non-programmable calculator is allowed.

1. (a) List the two essentials and two nonessentials in the problem to develop a model to obtain good estimates for future demand so as to help the soft drink company make right decisions.
(b) Assume that the return distribution of security is as given follows :

| Possible return | Associated <br> Probability |
| :---: | :---: |
| 0.01 | 0.2 |
| 0.07 | 0.2 |
| 0.08 | 0.3 |
| 0.1 | 0.1 |
| 0.15 | 0.2 |

Find the standard deviation of the security.
2. Consider the data shown in table given below : 5

| $x$ | $y$ |
| :---: | :---: |
| 2 | 1 |
| 9 | 17 |
| 3 | 3 |
| 5 | 9 |
| 1 | 0 |

Use a best fit line to estimate the value of $y$ for $x=6$ and 8 .
3. Do the stability analysis of the following Prey-

Predator model under toxicant stress in which it is assumed that the predators are not
affected by the toxicant because they are generally strong :

$$
\begin{aligned}
& \frac{d \mathrm{~N}_{1}}{d t}=r_{0} \mathrm{~N}_{1}-r_{1} \mathrm{CO} \mathrm{~N}_{1}-b \mathrm{~N}_{1} \mathrm{~N}_{2} \\
& \frac{d \mathrm{~N}_{2}}{d t}=-d_{0} \mathrm{~N}_{2}+\beta_{0} b \mathrm{~N}_{1} \mathrm{~N}_{2} \\
& \frac{d \mathrm{C}_{0}}{d t}=k_{1} \mathrm{P}-g_{1} \mathrm{C}_{0}-m_{1} \mathrm{C}_{0}
\end{aligned}
$$

where $\mathrm{N}_{1}(0) \geq 0, \mathrm{~N}_{2}(0) \geq 0, \mathrm{C}_{0}(0)=0$.
4. (a) Differentiate between the following terms : 2
(i) Linear and Non-linear models
(ii) Static and Dynamic models
(b) For the equation :

$$
\frac{d c}{d t}=\lambda c, \lambda=\mathrm{constant}
$$

If the tumour cells in a particular organ of a human body are $5 \times 10^{3}$, their growth increases upto $7.2 \times 10^{5}$ within five days. Find the value of $\lambda$. 3
5. Obtain the optimal solution of the following transportation problem :

|  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $a_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{1}$ | 7 | 3 | 4 | 2 |
| $\mathrm{O}_{2}$ | 2 | 1 | 3 | 3 |
| $\mathrm{O}_{3}$ | 3 | 4 | 6 | 5 |
| $b_{j}$ | 4 | 1 | 5 |  |

$a_{i}$ 's and $b_{j}$ 's represent supplies and requirements in a real situation and the elements of the matrix represent the corresponding casts.
6. Four counters are being run on the frontier of a country to check the passports and necessary papers of the tourists. The tourists choose a counter at random. If arrivals are Poisson at the rate $\lambda$ and the service time is exponential with parameter $\frac{\lambda}{2}$, what is the steady state average queue at each counter?

