M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

December, 2021

MMTE-001 : GRAPH THEORY

Time : 2 hours | *Maximum Marks : 50*
(Weightage : 50%)

Note : *Question no.* ***1*** *is* ***compulsory****. Answer any* ***four*** *questions from Q. Nos. 2 to 7. Use of calculators is not allowed.*

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1. State whether the following statements are *true* or *false*. Justify your answers with a short proof or a counter-example. *5×2=10*

(a) (4, 1, 1) is a graphic sequence.

(b) The complement of a tree is a forest.

(c) A 3-regular graph either has a cut-vertex or a cut-edge.

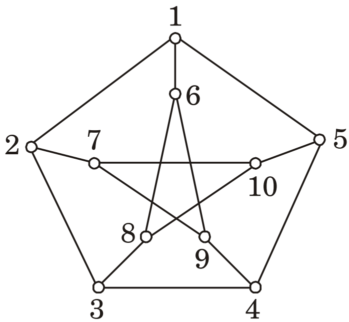
(d) $K_{20,\,22}$ is Eulerian.

(e) A k-chromatic graph has at least $\frac{k(k-1)}{2}$ edges.

2. (a) If P and P′ are two edge-disjoint paths having at least two common vertices, then show that $P \cup P'$ has a cycle. *2*

(b) Compute the number of perfect matchings in a complete graph on 2n vertices. *4*

(c) Prove that the following graphs are isomorphic. *4*



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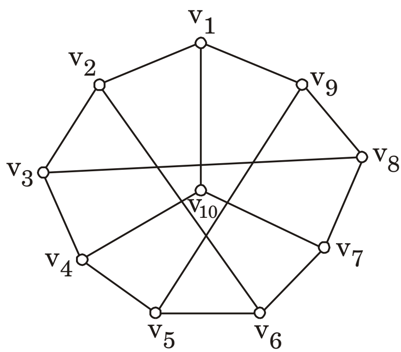

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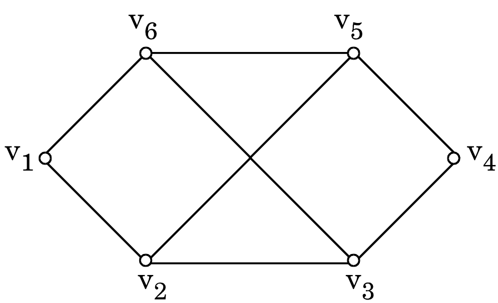
3. (a) Draw an Eulerian graph G such that $3 \leq \delta(G) - \Delta(G) \leq 4$. *2*

(b) Verify the König-Egeváry theorem for the following graph : *3*



(c) Prove that if a graph has no odd cycles, then it is bipartite. *5*

4. (a) If $(d_1, d_2, ..., d_n)$ is a graphic sequence, then so is $\left(d_1^2, d_2^2, ..., d_n^2\right)$. True or false ? Justify. *3*

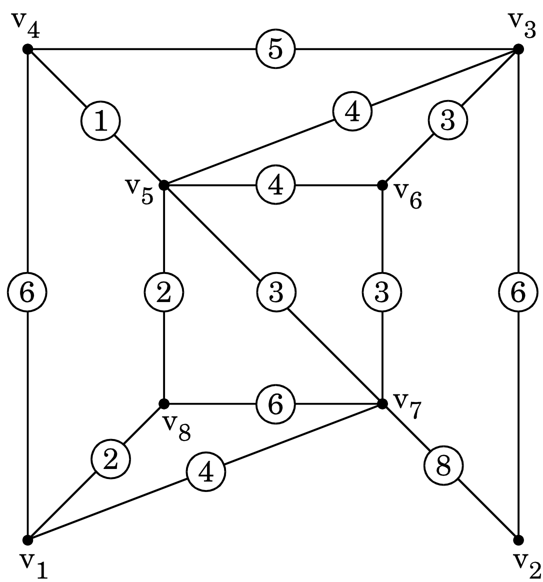
(b) Draw a tree T with at least 5 vertices for each of the following : *4*

(i) 2 rad (T) = diam (T)

(ii) diam (T) < 2 rad (T)

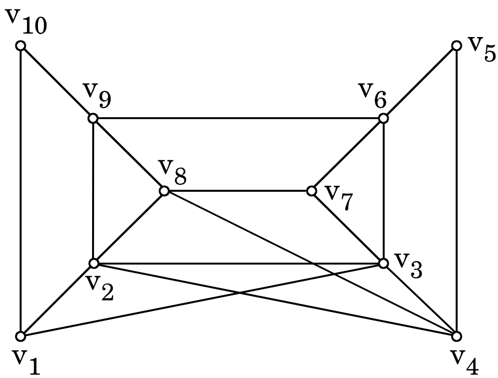
(c) Every Hamiltonian graph is 2-connected. Prove or disprove. *3*

5. (a) Find a minimum-weight spanning tree in the following graph, using Prim's algorithm. *5*



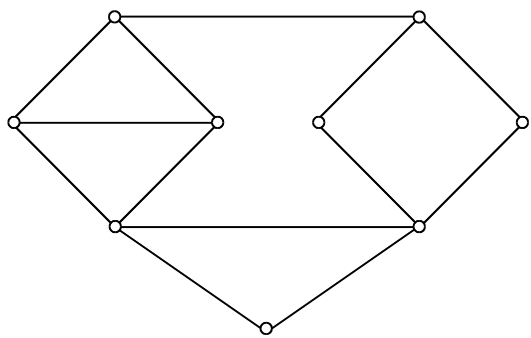
(b) State and prove Ore's theorem. *5*

6. (a) Find the chromatic number of the following graph. *3*



(b) Let G be a planar graph having at least 3 vertices and no 3-cycles. Show that $m(G) \leq 2n(G) - 4$. *3*

(c) Draw the dual of the following planar graph.



Does the dual have any cut-vertex or cut-edge ? Justify your answer. *4*

7. (a) Let G be a connected graph with blocks $B_1, B_2, ..., B_k$. Show that *4*

$$n(G) = \sum_{i=1}^{k} n(B_i) - k + 1.$$

(b) Show that the Peterson graph has its edge-connectivity equal to its vertex-connectivity. *3*

(c) Define a flow on the following network, having value at least 5. *3*