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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

December, 2021

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Attempt any **five** questions. Use of scientific non-programmable calculator is allowed.*

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1. (a) Find the number of covariances needed for evaluation of 200 securities using the Markowitz model. Also calculate the total number of pieces of information needed. *2*

(b) Suppose the population of fishes satisfy exponential growth model with an increase by 2·0% in an hour. If the initial population is 10,000, then find the population after 4 hours. How much time is required by the population to grow to triple of the initial size ? *3*

2. Ships arrive at a port at the rate of one in every 4 hours, with exponential distribution of inter-arrival times. The time a ship occupies a berth for unloading has exponential distribution with an average of 10 hours. If the average delay of ships waiting for berths is to be kept below 14 hours, how many berths should be provided at the port ? *5*

3. The yearly fluctuations in the groundwater table are believed to be dependent on the annual rainfall and the volume of water pumped out from the basin. The data collected on these variables for four consecutive years is given below :

Water table (in cm)	Annual rainfall (in cm)	Groundwater volume pumped out (in cm^3)
10	3	7
9	4	8
7	5	9
4	7	7

Use the method of least squares to find a linear regression equation that best fits the data. *5*

4. Consider the discrete time population model given by

$$N_{t+1} = \frac{r\,N_t}{1 + \left(\frac{N_t}{K}\right)^b}, \text{ for a population } N_t,$$

where K is the carrying capacity of the population, r is the intrinsic growth rate and b is a positive parameter. Determine the non-negative steady-state and discuss the linear stability of the model for $0 < r < 1$. Also find the first bifurcation value of the parameter r. *5*

5. (a) The deviation g(t) of a patient's blood glucose concentration from its optimal concentration satisfies the differential equation

$$\frac{d^2g}{dt^2} + 3\alpha\,\frac{dg}{dt} + 16\alpha^2\,g = 0,$$

for α being a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time t is measured in minutes. Identify the type (over-damped, under-damped or critically-damped) of this differential equation. Find the condition on α for which the patient is normal. *3*

(b) Let the returns of three securities A, B and C be 30%, 20%, and 15% respectively with $\sigma_A = 5$, $\sigma_B = 6$, $\sigma_C = 7$, $\sigma_{AB} = \sigma_{AC} = 16$ and $\sigma_{BC} = 10$. Find the standard deviation σ_P of the portfolio $P = (0{\cdot}4, 0{\cdot}1, 0{\cdot}5)$. *2*

6. A company has three factories F_1, F_2 and F_3, that supply to three markets M_1, M_2 and M_3. The transportation costs from each factory to each market are given in the table.

Capacities a_j's of the factories and the market requirements b_j's are shown below. Find the minimum cost of transportation. *5*

	M_1	M_2	M_3	a_j
F_1	7	3	4	2
F_2	2	1	3	3
F_3	3	4	6	5
b_j	4	1	5	