No. of Printed Pages : 6 **MMT-007**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M. Sc. (MACS)

Term-End Examination

Dec., 2021

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Question **No. 1** is **compulsory**.*

*(ii) Attempt any **four** questions out of Q. Nos. **2** to **7**.*

(iii) Use of scientific/non-programmable calculator is allowed.

1. State whether the following statements are True *or* False. Justify your answer with the help of a short proof or a counter-example : 5×2=10

(a) Initial value problem :

$$\frac{dy}{dx} = \frac{y-1}{x}$$

$y(0) = 1$ has a unique solution.

(b) $L[t^2 \cos(at)] = \dfrac{2s(s^2 - 3a)}{(s^2 + a^2)^2}$.

(c) The second order Runge-Kutta method when applied to IVP $y' = -100\, y,\ y(0) = 1$ will produce stable results for $0 < h < \dfrac{1}{50}$.

(d) If Fourier cosine transform of $f(x)$ is :

$$F_c(n) = \frac{\cos\left(\frac{2n\pi}{3}\right)}{(2n+1)^2}$$

where $0 \le x \le 1$, then :

$$f(x) = 1 + 2\sum_{n=1}^{\infty} \frac{\cos\left(\frac{2n\pi}{3}\right)}{(2n+1)^2} \cos n\pi x.$$

(e) Finite element Galerkin method is a weighted residual method and requires the variational form of the given differential equation.

2. (a) Show that :

$$\int_0^\infty e^{-st} \, L_n(t) \, dt = \frac{1}{s}\left(1 - \frac{1}{s}\right)^n$$

where $L_n(t)$ is a Laguerre polynomial. 4

(b) Using Schmidt method with $\lambda = \frac{1}{4}$, find the solution of initial value problem :

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

with $u\ (0,\ t) = 0 = u\ (1,\ t)$ and $u\ (x, 0) = \sin \pi x,\ 0 \le x \le 1$ with $h = \frac{1}{3}$. 4

(c) Using the substitution $z = \sqrt{x}$, reduce the given equation to Bessel equation and hence find its solution : 2

$$xy'' + y' + \frac{y}{4} = 0$$

3. (a) Find the power series solution of the equation :

$$x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + (x^2 + 2)\ y = 0$$

about its singular point. 5

(b) Construct Green's function for the following boundary value problem :

$$\frac{d^2 y}{dx^2} + 9y = 0$$

with $y\ (0) = y\ (1) = 0$. 5

4. (a) Find the solution of $\nabla^2 u = 0$ in R subject to R : triangle $0 \le x \le 1$, $0 \le y \le 1$, $0 \le x + y \le 1$ and $u\ (x, y) = x^2 - y^2$ on the boundary of the triangle. Assume $h = \frac{1}{4}$ and use five-point formula. 5

(b) Show that the method :

$$y_{i+1} = \frac{4}{3} y_i - \frac{1}{3} y_{i-1} + \frac{2h}{3} y'_{i+1}$$

is absolutely stable when applied to the equation $y' = \lambda y,\ \lambda < 0$. 3

(c) Evaluate $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$, using convolution theorem. 2

5. (a) Given $\frac{dy}{dx} = \frac{1}{2}(1+x^2)\,y^2$ and $y\,(0) = 1$, $y\,(0.1) = 1.06$, $y\,(0.2) = 1.12$, $y\,(0.3) = 1.21$, evaluate $y\,(0.4)$ using Milne's predictor-corrector method. Use one corrector iteration. 6

(b) For Bessel's function, show that : 4

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x.$$

6. (a) Find the solution of the initial boundary value problem :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 \le x \le 1$$

$u\,(x, 0) = \sin\,(\pi x),\ 0 \le x \le 1,\ \frac{\partial u}{\partial t}(x, 0) = 0$, $0 \le x \le 1$, $u\,(0, t) = u\,(1, t) = 0$, $t > 0$.

by using second order explicit method with $h = \frac{1}{3}, r = \frac{1}{2}$. Integrate for two time steps. 6

(b) Find $L^{-1}\left\{\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)\right\}$, L^{-1} being Laplace inverse transform. 4

7. (a) Heat conduction equation $u_t = u_{xx}$ is approximated by the method :

$$u_m^{n+1} - u_m^{n-1} = \frac{2k}{h^2}\,\delta_k^2 u_m^n$$

Find the order of the method and investigate the stability of this method using Von Neumann method. 5

(b) Using second order finite difference method with $h = \frac{1}{2}$, obtain the system of equations for y_0, y_1 and y_2 for solving the boundary value problem : 5

$$y'' - 5y' + 6y = 3$$

with $y\,(0) - y'\,(0) = -1$ and $y\,(1) + y'\,(1) = 1$.