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MMT-005

**M. Sc. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER**

SCIENCE) M. Sc. (MACS)

Term-End Examination

December, 2021

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ Hours

Maximum Marks : 25

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **three** questions from Q.*

Nos. 2 to 5.

(iii) *Use of calculator is **not** allowed.*

1. State, giving reasons whether the following statements are True or False : 10

(a) $\{z : |\operatorname{Im} z| \leq (\operatorname{Re} z)^2\}$ is a region.

(b) $w = z^n$ $n \in \mathbb{N}$ is conformal at $z = 0$.

(c) The radius of convergence of any power series is positive.

(d) The maximum value of the function $f(z) = e^z + 8$ in $|z - 1| \leq 1$ is $e + 8$.

(e) The function $f(z) = \sin z$ is bounded in the region $\{z \in \mathbb{C} \mid \operatorname{Re}(z) > 0\}$.

2. (a) Find the Laurentz series expansion of the function : 3

$$f(z) = \frac{z}{z^2 - 3z + 2}$$

in the annulus region $1 < |z| < 2$.

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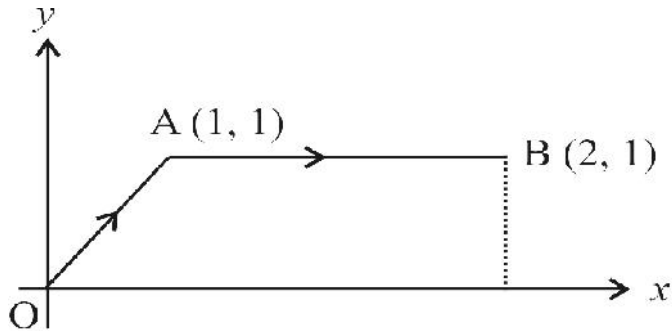
(b) Find the zeroes and singularities of the

$$\text{function } f(z) = \frac{z}{2 \sin^2 z - 1} \text{ in } |z| \leq 1. \quad 2$$

3. (a) Find the harmonic conjugate of the function : 2

$$u(x, y) = x^3 - 3x^2y - 3xy^2 + y^3$$

(b) Evaluate $\int_C (z^2 + 1) dz$, where OAB is the path as shown in the figure below : 2



(c) Define a branch of a multivalued function. 1

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4. (a) Find the image of the strip $x < 0$, $-1 < y < 1$ under the mapping $w = 2z - 1$. Sketch the strip and its image. 3

(b) Find all possible values of $(-i)^i$. 2

5. (a) Evaluate : 3

$$\int_{|z|=1} \frac{dz}{\cos z - \sin z}$$

(b) If $f(z)$ is an entire function such that :

$$|f(z)| \leq e^{\operatorname{Im}(z)} \quad \forall z \in \mathbb{C},$$

show that $f(z) = ae^{-iz}$, where $|a| \leq 1$. 2

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P. T. O.