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M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M. Sc. (MACS)

Term-End Examination

December, 2021

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Note : *Question No. **5** is **compulsory**. Answer any **three** questions from Q. Nos. **1** to **4**. Calculators are not allowed.*

1. (a) Let the matrix of a linear operator T with respect to an ordered basis $\{u_1, u_2, u_3\}$ of $\mathbf{R}^3$ be :

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Find the matrix of T with respect to the basis $\{u_1 + u_2 + u_3, u_1 - u_2 + u_3, u_2 + u_3\}$. 3

(b) What are the singular values of the matrix 2

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}?$$

2. (a) Find a unitary matrix P so that the matrix $P^{-1}AP$ is a diagonal matrix, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. 3

(b) Find all integers t so that the matrix $\begin{bmatrix} 4 & t & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ is diagonalizable. 2

3. (a) Write all possible Jordan canonical forms for a 4×4 matrix whose only distinct eigen values are 1 and 2, the geometric multiplicity of 1 is two and the minimal polynomial is of degree 3. 2

(b) Let :

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 2 & 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Show that the system $Ax = b$ is inconsistent. Find a least squares solution for this system $Ax = b$. 3

4. (a) Evaluate $\exp\left(A^2\right)$, if $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. 2

(b) Find the QR decomposition of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. 3

5. Which of the following statements are true. and which are not ? Give reasons for your answers : 10

(i) The sum of two eigen values of a linear operator T is also an eigen value of T.

(ii) If A is an $n \times n$ diagonalizable matrix and I_n is the $n \times n$ identity matrix, then $I_n + A^3$ is also diagonalizable.

(iii) If A is an $n \times n$ matrix, with all eigen values 1, then A is the identity matrix.

(iv) There is no unitary matrix whose one of the entries is 3.

(v) If A is a normal operator, A^* is also a normal operator.

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