POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

February, 2021

MST-003 : PROBABILITY THEORY

Time : 3 hours *Maximum Marks : 50*

Note :

(i) *Question no.* ***1*** *is* ***compulsory****.*

(ii) *Attempt any* ***four*** *questions from the remaining questions.*

(iii) *Use of scientific (non programmable) calculator is allowed.*

(iv) *Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.*

(v) *Symbols have their usual meanings.*

1. State whether the following statements are *True* or *False*. Give reasons in support of your answers. *5×2=10*

(a) A coin is weighted so that head is twice as likely to appear as tail. Then Prob[Head in any trial] = $\frac{2}{3}$.

(b) If random variable X has the probability density $f(x) = k \cdot e^{-3x}$ for $x > 0$, then the value of k is $\frac{1}{3}$.

(c) If random variable X follows $\beta\left(18, \frac{1}{3}\right)$, then the value of $\beta_1 = \frac{1}{36}$.

(d) For a certain random variable X following Normal distribution, the fourth central moment about 50 is 48. Then the variance of the X is 4.

(e) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, and $P(C) = \frac{1}{6}$ and events A, B and C are mutually exclusive, then $P(A \cup B \cup C) = \frac{3}{4}$.

2. A man has three coins A, B and C. Coin A is unbiased. The probability that a head will result when coin B is tossed is $\frac{2}{3}$. The probability that a head will result when coin C is tossed is $\frac{1}{3}$. If one of the coins, chosen at random, is tossed three times, and results in two heads and one tail, find the probability that the chosen coin was (i) A, (ii) B, and (iii) C. *10*

3. (a) A random variable X takes values – 1, 0, 2 and 3 with probabilities 0·27, 0·12, 0·26 and 0·35, respectively. Then show that $E(X + 1) - E(X) = 1$ using above information. *4*

(b) Given the following bi-variate probability distribution :

Y \ X	–1	0	1
0	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{15}$
1	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{1}{15}$
2	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$

Obtain (i) marginal distribution of X and Y, and (ii) condition distribution of X given Y = 2. *6*

4. (a) A perfect die is thrown a large number of times in sets of 8. The occurrence of a 5 or a 6 is called a success. In what proportion of the sets would you expect 3 successes ? *5*

(b) If a lot of 40 bulbs contains 3 defectives, and five bulbs are selected at random without replacement, what is the probability that exactly 1 defective is found ? Also obtain the mean of the distribution. *5*

5. (a) If X is a normal variate with mean 30 and standard deviation 5, then obtain : *6*

(i) $P(26 \leq X \leq 40)$,

(ii) $P(X \geq 45)$, and

(iii) $P(|X - 30| > 5)$.

(b) If X is uniformly distributed with mean 4 and variance 48, find $P[\overline{X} < 2]$. *4*

6. (a) For the given p.d.f. : *1+1+2+1*

$$f(x) = \begin{cases} 2\,e^{-kx}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k.

(ii) Identify the distribution.

(iii) Find mean and variance of the distribution under consideration.

(iv) Obtain the cumulative distribution function.

(b) A machine is known to produce 3% defective items. What is the probability that at least 5 items are to be examined in order to get 2 defective items ? *5*

7. (a) Two dice are thrown. Find the probability that the sum of the numbers on the faces is (i) 7 or 8, (ii) more than 9. *4*

(b) Distinguish between *6*

(i) a discrete random variable and a continuous random variable, with examples of each.

(ii) probability mass function and probability density function.