M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

February, 2021

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no.* ***7*** *is* ***compulsory****. Answer any* ***four*** *out of the remaining six (Q. 1 to 6). Use of calculators is not allowed.*

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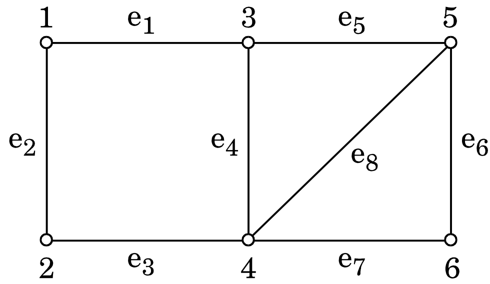
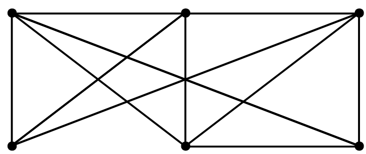
1. (a) Let V be the collection of all two-element subsets of a five-element set. Consider graph G with V as vertex set and E as edge set, where u, v $\in$ E if u, v $\in$ V, and u and v are disjoint. Draw G and identify the graph. Check whether this graph is bipartite or not. *5*

(b) Write down the adjacency matrix and incidence matrix of the following graph : *5*



2. (a) Prove that a graph is bipartite if and only if it has no odd cycle. *5*

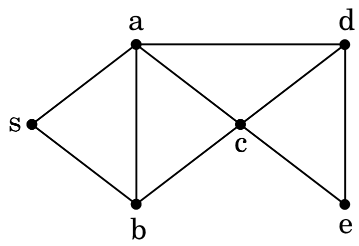
(b) Check whether the following graph is Eulerian or not. Is it Hamiltonian ? Justify your answer. *5*



3. (a) Prove that the non-negative integers $d_1, d_2, ..., d_n$ are the vertex degrees of some simple graph only if $\sum_{i=1}^{n} d_i$ is even. *2*

(b) Let T be a tree with k-edges and G be a simple graph with minimum degree at least k. Prove that T is a subgraph of G. *5*

(c) Use the BFS algorithm to find shortest paths from s to every other vertex. *3*



4. (a) Define a maximal matching and a maximum matching.
Give an example of a graph having a maximal matching M such that M is not a maximum matching. *4*

(b) State and prove Hall's theorem on matchings. *6*

5. (a) If G is a 3-regular graph, then prove that $\kappa(G) = \kappa'(G)$. *4*

(b) Find the chromatic number of Petersen graph. Justify your answer. *4*

(c) If G is a k-critical graph, then prove that $\delta(G) \geq k - 1$. *2*

6. (a) If G is a simple planar graph of order at least 3, then prove that $e(G) \leq 3\, n(G) - 6$. Is the converse true ? Justify your answer. *5*

(b) If G is a simple graph with $n \geq 3$ vertices, then prove that G is Hamiltonian if $\delta(G) \geq \frac{n}{2}$. *5*

7. State whether the following statements are *true* or *false*. Give reasons for your answers. *5×2=10*

(a) If $G \cong H$, then $\overline{G} \cong \overline{H}$.

(b) Deletion of a vertex from a tree always produces a tree.

(c) Every k-regular graph has a perfect matching.

(d) Every 2-colourable graph is planar.

(e) There exists a non-bipartite graph for which the maximum size of a matching is equal to the cardinality of a minimum vertex cover.