No. of Printed Pages : 5 **MMTE-005**

# M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) (MACS)

## Term-End Examination

## December, 2020

### MMTE-005 : CODING THEORY

*Time : 2 Hours* *Maximum Marks : 50*

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***Note :*** *(i) Answer any **four** questions from question no. **1** to **5**.*

*(ii) Question **No. 6** is compulsory.*

*(iii) Use of calculator is **not** allowed.*

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1. (a) Define and give an example each of the following : 5

(i) Generator matrix of a linear code,

(ii) Minimum weight of a linear code.

Justify your choice of examples.

(b) Are the $\mathbb{Z}_4$ linear codes with generator matrices : 5

$$G_1 = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

and $$G_2 = \begin{bmatrix} 0 & 2 & 3 & 3 \\ 3 & 0 & 3 & 3 \\ 0 & 1 & 3 & 3 \end{bmatrix}$$

monomially equivalent ? Why, or why not ?

2. (a) Check whether or not a [23, 12, 7] binary code is perfect. 3

(b) Give an example, with justification, of a QR code of length 7 over $\mathbb{F}_8$. 2

(c) Write the generator matrix of the binary (7, 4) cyclic code with generator polynomial $x^3 + x + 1$. Also find the parity check matrix of this code. 5

3. (a) Draw the Tanner graph of the code C with parity check matrix : 5

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

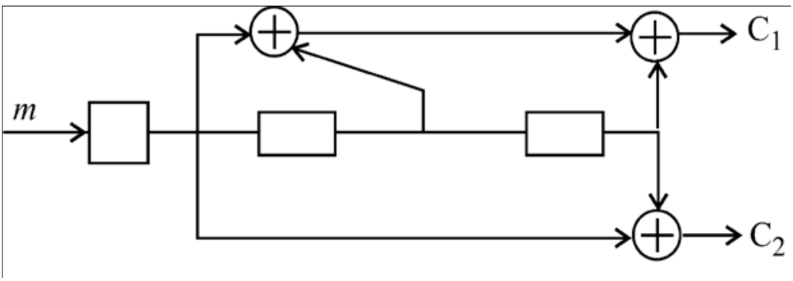
Does the graph contain a cycle ? Justify your answer.

(b) Show that the polynomial :

$$f(x) = x^3 + x + 1$$

is irreducible over $\mathbb{F}_2$. How many elements does $\dfrac{\mathbb{F}_2[x]}{\langle f(x) \rangle}$ have, and why ? Further, find the inverse of $\alpha \in \dfrac{\mathbb{F}_\alpha[x]}{\langle f(x) \rangle}$, where $\alpha$ is a root of $f(x)$. 5

4. (a) Find the convolutional code for the message 11011, for the convolutional encoder given below : 4




(b) Let C be the narrow sense binary BCH code of designed distance $\delta = 5$, which has a defining set T = {1, 2, 3, 4, 6, 8, 9, 12}. Let $\alpha$ be a primitive 15th root of unity, where

$\alpha^4 = 1 + \alpha$, and let the generator polynomial of C be : 6

$$g(x) = 1 + x^4 + x^6 + x^7 + x^8$$

If $y(x) = x^9 + x^8 + x^5 + x^4 + x^3 + 1$

is received, find the transmitted code word. You can use the following table :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0000 | 0 | 1000 | $\alpha^3$ | 1011 | $\alpha^7$ | 1110 | $\alpha^{11}$ |
| 0001 | 1 | 0011 | $\alpha^4$ | 0101 | $\alpha^8$ | 1111 | $\alpha^{12}$ |
| 0010 | $\alpha$ | 0110 | $\alpha^5$ | 1010 | $\alpha^9$ | 1101 | $\alpha^{13}$ |
| 0100 | $\alpha^2$ | 1100 | $\alpha^6$ | 0111 | $\alpha^{10}$ | 1001 | $\alpha^{14}$ |

5. (a) Let C be the [5, 2] binary code generated by $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$. Find the weight distribution of C. Find the weight distribution of $C^{\perp}$ by using MacWilliams identity. 4

(b) Let C be any $\left[n, \dfrac{n-1}{2}\right]$ cycle code over $\mathbb{F}_q$. Prove that C is self-orthogonal if and only if C is an even-like duadic code whose splitting is given by $\mu_{-1}$. 6

6. Which of the following statements are true, and which are false ? Give reasons for your answers. Marks will only be given for valid reasons : 10

   (i) If a code is self-orthogonal, it is self-dual.

   (ii) If the syndrome of the received code word is zero, then there is no error in the transmission.

   (iii) [1 0] generates a Reed-Muller code.

   (iv) $\mathbb{F}_2 \times \mathbb{F}_3$ is a finite field.

   (v) The generator polynomial of a Reed-Solomon code over $\mathbb{F}_q$ splits into linear factors over $\mathbb{F}_q$.