No. of Printed Pages : 3

MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

February, 2021

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no.* ***5*** *is* ***compulsory****. Answer any* ***three*** *questions from Questions no. 1 to 4. Use of calculators is* ***not*** *allowed.*

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1. (a) Let $\mathcal{B} = \{u_1, u_2, u_3\}$ be an ordered basis of $\mathbf{R}^3$ and let the matrix of a linear operator T on $\mathbf{R}^3$ with respect to this basis be

$$[T]_{\mathcal{B}} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Find the matrix of T with respect to the basis $\{u_1 + u_2 + u_3, u_2 + u_3, u_1 + u_3\}$. *2*

(b) Obtain the QR-decomposition for the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$ *3*

2. (a) Is $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ diagonalisable ? If yes, find an invertible matrix P so that $P^{-1}AP$ is a diagonal matrix. If A is not diagonalisable, find the Jordan canonical form of A. *3*

(b) Obtain the singular values of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$. *2*

3. (a) Obtain the Jordan canonical form for the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ *$2\frac{1}{2}$*

(b) Find the square root of $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. *$2\frac{1}{2}$*

4. (a) Find a least squares solution of Ax = y, where $A = \begin{bmatrix} 2 & -2 \\ 1 & 3 \\ 1 & 1 \end{bmatrix}$, $y = \begin{bmatrix} 8 \\ 0 \\ 1 \end{bmatrix}$. *2*

(b) In a city, the monkey (M) and dog (D) populations are governed by the following equations :

$M_{k+1} = 0{\cdot}5\, M_k + 0{\cdot}4\, D_k$

$D_{k+1} = -\,0{\cdot}104\, M_k + 1{\cdot}1\, D_k$

What is the ratio of the two populations in the long run ? *3*

5. Which of the following statements are *True* and which are not ? Give reasons for your answers in the form of a short proof or a counter-example. *5×2=10*

(a) If A is a non-zero 2×3 matrix, then there is a 3×2 non-zero matrix B such that $AB = 0$.

(b) If A is an $n \times n$ nilpotent matrix, then the product of the eigenvalues of $I_n + A$ is non-zero, where I_n is the $n \times n$ identity matrix.

(c) There is no normal matrix having one of the entries as 2.

(d) If A is positive definite, then A^{-1} is positive definite.

(e) Every upper triangular matrix is diagonalisable.
