BACHELOR OF COMPUTER APPLICATIONS (B. C. A.) (Revised)

Term-End Examination

December, 2020

BCS-012 : BASIC MATHEMATICS

Time : 3 Hours *Maximum Marks : 100*

Note : *Question number **1 is compulsory**. Attempt any **three** questions from the remaining questions.*

1. (a) Show that : 5

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a).$$

(b) Use the principle of mathematical induction to prove : 5

$$\frac{1}{(1)(2)} + \frac{1}{(2)(3)} + \ldots\ldots + \frac{1}{(n)(n+1)} = \frac{n}{n+1},$$

where n is a natural number.

(c) Find the sum of n terms, for the series given below : 5

$$3 + 33 + 333 + \ldots\ldots\ldots\ldots .$$

(d) Evaluate : 5

$$\lim_{x \to 0} \frac{\sqrt{1+2x}-\sqrt{1-2x}}{x}.$$

(e) Evaluate : 5

$$\int \frac{dx}{\sqrt{x}+x}.$$

(f) If $y = ax + \frac{b}{x}$, show that : 5

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

(g) If 1, ω and ω^2 are the cube roots of unity, show that : 5

$$(1+\omega+\omega^2)^5 + (1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5 = 32.$$

(h) Find the value of λ for which the vectors $\vec{a} = \hat{i} - 4\hat{j} + \hat{k}$; $\vec{b} = \lambda\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + 3\hat{k}$ are coplanar. 5

2. (a) Solve the following system of equations, using Cramer's rule : 5

$$x + 2y + 2z = 3;$$

$$3x - 2y + z = 4;$$

$$x + y + z = 2.$$

(b) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A - 5I_3 = 0$.

Hence find A^{-1} and A^3. 10

(c) If $A = \begin{bmatrix} 3 & 4 & -5 \\ 1 & 1 & 0 \\ 1 & 1 & 5 \end{bmatrix}$, show that A is row equivalent to I_3. 5

3. (a) Solve the equation $2x^3 - 15x^2 + 37x - 30 = 0$, given that the roots of the equation are in A. P. 5

(b) If 1, ω and ω^2 are cube roots of unity, show that $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11}) = 49$. 5

(c) Use De-Moivre's theorem to find $\left(\sqrt{3}+i\right)^3$ .5

(d) Find the sum of an infinite G. P., whose first term is 28 and fourth term is $\frac{4}{49}$. 5

4. (a) Determine the values of x for which the following function is increasing and decreasing : 5

$$f(x) = (x-1)(x-2)^2$$

(b) Find the length of the curve $y = 2x^{3/2}$ from the point (1, 2) to (4, 16). 5

(c) If $\vec{a}+\vec{b}+\vec{c}=10$, show that : 5

$$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}.$$

(d) Solve : 5

$$\frac{2x-5}{x+2}<5,\ x\in \mathrm{R}.$$

5. (a) A man wishes to invest at most ₹ 12,000 in Bond-A and Bond-B. He must invest at least ₹ 2,000 in Bond-A and at least ₹ 4,000 in Bond-B. If Bond-A gives return of 8% and Bond-B gives return of 10%, determine how much money, should be invested in the two bonds to maximize the returns. 10

(b) Find the points of local maxima and local minima of the function $f(x)$, given below : 5

$$f(x)=x^3-6x^2+9x+100.$$

(c) Show that 7 divides $2^{3n}-1$, $\forall n\in \mathbf{N}$ i. e. set of natural numbers, using mathematical induction. 5