

POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

December, 2019

MST-003 : PROBABILITY THEORY

Time : 3 hours

Maximum Marks : 50

Note : (i) Question No. 1 is compulsory.

(ii) Attempt any four questions from the remaining question Nos. 2 to 7.

(iii) Use of Scientific (non-programmable) calculator is allowed.

(iv) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.

(v) Symbols have their usual meanings.

1. State whether the following statements are True or False. Give reasons in support of your answer. 5x2=10

(a) If $P(A) = P(B) = P(A \cap B)$ then $P[(A \cap \bar{B}) \cup (\bar{A} \cap B)] = 1$.

(b) If X is uniformly distributed with mean 1 and variance $\frac{4}{3}$, then $P(X < 0) = \frac{1}{4}$.

(c) A random variable X has the following probability distribution :

| | | | | | | | | |
|--------|-----|-----|------|------|------|-------|--------|------------|
| x | : 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $P(x)$ | : 0 | k | $2k$ | $2k$ | $3k$ | k^2 | $2k^2$ | $7k^2 + k$ |

Then the value of k is 0.10.

(d) A continuous random variable X has a pdf $f(x) = kx^2e^{-x}$ where $x > 0$ with mean and variance '3'.

(e) The probability of the 5th hit in 10th draw by a person is 0.72, if probability of hitting the target at any trial is 0.5.

2. (a) An urn contains 5 white and 3 green balls and another urn contains 3 white and 7 green balls. Two balls are chosen at random from the first urn and put into the second urn. Then the ball is drawn from the second urn. What is the probability that it is a white ball? 7

(b) Two cards are drawn from a pack of 52 cards, find the probability that draw includes an ace and a ten. 3

3. The Joint probability density function of a bivariate random variable (X, Y) is
 $f_{X,Y}(x, y) = k(x+y), 0 < x < 2; 0 < y < 2$ 2+4+1+3
 $= 0$, otherwise
 where k is a constant.
- (a) Find the value of k .
- (b) Find the marginal probability density functions of X and Y .
- (c) Are X and Y independent ?
- (d) Find the conditional distributions $f_{Y/X}(y/x)$ and $f_{X/Y}(x/y)$.
4. (a) An item is produced in large numbers. The machine is known to produce 5% defectives. A quality control inspector is examining the items by taking them at random. What is the probability that atleast 4 items are examined in order to get 2 defectives ? 5
- (b) A lot consisting of 100 fuses, is inspected by the following procedure. Five of these fuses are chosen at random and tested; if 4 or more work at the correct amperage, the lot is accepted. If there are 20 defective fuses in the lot, find the probability of acceptance. 5
5. (a) A company has installed 10,000 electric lamps in a metro. If these lamps have an average life of 1000 burning hours with a standard deviation of 200 hours. Assuming normality, what number of lamps might be expected to fail (i) in the first 800 burning hours; (ii) between 800 and 1200 burning hours. 3+3
- (b) A system contains a certain type of component whose life time X is exponentially distributed with mean 5 years. Find the probability that the component has life greater than 5 years. 4
6. (a) Show that the probability of getting 3 at least once in 4 throws of a die is greater than the probability of getting a double six at least once in 24 throws with two dice. 5
- (b) A random variable X has the following probability mass function : 5
- | | | | | | | | | | |
|----------|-----|------|------|------|------|-------|-------|-------|-------|
| $x :$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $P(x) :$ | a | $3a$ | $5a$ | $7a$ | $9a$ | $11a$ | $13a$ | $15a$ | $17a$ |
- (i) Determine the value of a .
- (ii) Find $P(X < 3)$.
- (iii) Find the distribution function of X .
7. (a) The mean and variance of a binomial variate X with parameters (n, p) are 16 and 8. Find (i) $P(x=0)$, (ii) $P(x=1)$ and (iii) $P(x < 2)$. 5
- (b) The daily consumption of milk in a city, in excess of 20,000 litres, is approximately distributed as a Gamma variate with parameters $r=2$ and $\lambda=1/10,000$. The city has a daily stock of 30,000 litres. What is the probability that the stock is sufficient on a particular day ? 5