No. of Printed Pages : 4 **MMTE-005**

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination, 2019

MMTE-005 : CODING THEORY

Time : 2 Hours] [Maximum Marks : 50

[Weightage : 50%

Note : Answer **any four** questions from Question No. 1 to 5. Question No. 6 is **compulsory**. Calculator is **not** allowed.

1. (a) Let C_1 be the [4, 3, 2]-binary linear code generated by $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, and let C_2 be the [4, 1, 4] - binary linear code generated by [1111]. Give the parameters with justification and find a generator matrix of the $(\underline{u} / \underline{u} + \underline{v})$ construction of the codes C_1 and C_2. [3]

(b) Construct a generator matrix of the Reed-Muller code R(2, 4). [4]

(c) If a polynomial generator matrix of an (n, k) convolutional code C is basic and reduced, then prove that it is canonical. [3]

2. (a) Let $\mathbb{F}_q$ have characteristic p. Prove that $(\alpha + \beta)^p = \alpha^p + \beta^p$ for all $\alpha, \beta \in \mathbb{F}_q$. [2]

(b) Find gcd $(x^5 - x^4 + x + 1, x^3 + x)$ in $\mathbb{F}_3[x]$. [3]

(c) Construct a [13, 10, 3] BCH code over $\mathbb{F}_3$. [5]

3. (a) Construct the Tanner graph for the given parity check matrix H of an LDPC code. Further, does the code contain a cycle ? Justify your answer. [5]

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Find the generator matrix of the (7, 4) cyclic code generated by the polynomial $g(x) = 1 + x + x^3$. [5]

4. (a) Let C be a self-dual [24, 12, 8] doubly even binary code. Find the weight enumerator of C by using Mac-Williams identities. [8]

(b) Does there exist a QR code of length 17 over $\underset{\sim}{F}_{11}$? Give reasons for your answer. [2]

5. (a) Show that the $\underset{\sim}{Z}_4$-linear codes with generator matrices : [5]

$$G_1 = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \text{ and } G_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

are monomially equivalent.

(b) If $\underset{\sim}{x}, \underset{\sim}{y} \in \underset{\sim}{F}_2^n$, show that :

$$\text{wt}(\underset{\sim}{x} + \underset{\sim}{y}) = \text{wt}(\underset{\sim}{x}) + \text{wt}(\underset{\sim}{y}) - 2\text{wt}(\underset{\sim}{x} \cap \underset{\sim}{y}),$$

where $\underset{\sim}{x} \cap \underset{\sim}{y}$ is the vector in $\underset{\sim}{F}_2^n$ which has 1 in precisely those positions where $\underset{\sim}{x}$ and $\underset{\sim}{y}$ have 1.

Further, show that if C is a binary code with a generator matrix, each of whose rows are of even weight, then every codeword of C has even weight. [5]

6. Which of the following statements are true ? Give reasons for your answers in the form of a short proof or a counter example : [10]

(i) The code over F_3 with generator matrix

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 \end{bmatrix}$$ is self-orthogonal.

(ii) Every cyclic code is self-dual.

(iii) $x^3 - 1 + x$ is irreducible over F_5.

(iv) Two distinct codes can have the same generator matrix.

(v) The number of 3-cyclotomic cosets modulo 26 is 3.

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