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MMT-007

MASTER OF SCIENCE (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M. Sc. (MACS)

Term-End Examination

December, 2019

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No. 1 is compulsory. Answer any four questions out of Question Nos. 2 to 7. Use of scientific non-programmable calculator is allowed.*

1. State whether the following statements are true *or* false. Justify your answer with the help

of a short proof or a counter example. No marks are awarded without justification : 5×2=10

(a) For boundary value problem :

$$\frac{d^2y}{dx^2} - 9y = 3x$$

$$y(0) = y(1) = 0$$

the Green's function G (x, ξ) satisfies :

$$\frac{d^2G(x,\xi)}{dx^2} - 9G(x,\xi) = x.$$

(b) The order of the five-point finite difference formula for Poisson equation $\nabla^2 u = G(x, y)$ is $o(h^2)$.

(c) $\int_{-1}^{+1} P_2(x)\, dx = 1.$

(d) If F^{-1} denotes inverse Fourier transform, then :

$$F^{-1}\left[\frac{1}{\alpha^2 + 2\alpha + 5}\right] = \frac{1}{4} e^{-[2|x| + ix]}$$

(e) The interval of absolute stability of the Runge-Kutta method :

$$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_i + h_1 y_i + k_1)$$

is $-2 < \lambda h < 0$.

2. (a) Find the series solution, near $x = 0$, of the differential equation : 4

$$x^2 y'' + xy' + (x^2 - n^2)\,y = 0$$

where n is a non-zero real constant.

(b) Using the Fourier transforms, determine the solution of the equation : 6

$$\frac{\partial^4 z}{\partial x^4} + \frac{\partial^2 z}{\partial y^2} = 0 \quad (-\infty < x < \infty,\ y > 0)$$

satisfying the conditions :

(i) z and its partial derivatives tend to zero as $z \to \pm\infty$.

(ii) $z = f(x)$, $\frac{\partial z}{\partial y} = 0$ on $y = 0$.

3. (a) Solve the initial value problem : 6

$$y' = x^2 + \sqrt{y} + 1,\ y(0) = 1$$

upto $x = 0.6$, using Predictor-Corrector method :

$$\text{P} : y_{n+1}^{(p)} = y_n + \frac{h}{2}(3f_n - f_{n-1})$$

$$\text{C} : y_{n+1}^{(c)} = y_n + \frac{h}{12}[\,5f\,(x_{n+1}, y_{n+1}^{(p)}) + 8f_n - f_{n-1}]$$

with step length $h = 0.2$. Compute the starting value using Euler's method and perform two corrector iterations per step.

(b) If L^{-1} denote inverse Laplace transform, find $\text{L}^{-1}\left[\ln\left(1 + \frac{1}{s^2}\right)\right]$. 4

4. (a) Using second order method, solve the boundary value problem : 6

$$y'' = xy,\ y(0) + y'(0) = 1,\ y(1) = 1$$

with $h = \frac{1}{3}$.

(b) Find the Fourier cosine transform of: 4

$$f(x) = \begin{cases} x & , \ 0 < x < \frac{1}{2} \\ 1 - x \ , & \frac{1}{2} < x < 1 \\ 0 & , \ x > 1 \end{cases}$$

5. (a) Find the first three terms of the Legendre series for the function :

$$f(x) = \begin{cases} 0, & -1 < x \le 0 \\ x, & 0 < x < 1 \end{cases}$$

(b) For the initial value problem : 5

$$\frac{dy}{dx} = f(x, y), \ y(x_0) = y_0$$

derive the general multi-step method given by :

$$y_{i+1} = a_1 y_i + a_2 y_{i-1} + h b_0 y'_{i+1}$$

Find the truncation error and the order of the method. 5

6. (a) Determine the interval of absolute stability for the method : 6

$$y_{i+1} = y_{i-1} + \frac{h}{3}(7y'_i - 2y'_{i-1} + y'_{i-2})$$

when applied to the equation $y' = \lambda y$, $\lambda < 0$.

(b) Using Schmidt method with $\lambda = \frac{1}{6}$ and $h = 0.2$, find the solution of initial boundary value problem, subject to the given initial and boundary conditions : 4

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$u(x, 0) = \begin{bmatrix} 2x & \text{for } x \in \left[0, \frac{1}{2}\right] \\ -2x & \text{for } x \in \left[\frac{1}{2}, 1\right] \end{bmatrix}$$

and $u(0, t) = 0 = u(1, t)$.

Integrate for one time level.

7. (a) Express $f(x) = x^3 - 3x^2 + 2x$ in a series of the form $\sum_{n=0}^{\infty} a_n H_n(x)$, where $H_n(x)$ is the Hermite polynomial of degree n in x. 3

(b) Using Laplace transform method, solve the differential equation : 4

$$y'' + 2y' - 3y = 3,\ y(0) = 4,\ y'(0) = -7$$

(c) Prove that : 3

$$\int J_0(x)\, J_1(x)\, dx = -\frac{1}{2} J_0^2(x) + c.$$