106815

No. of Printed Pages : 2

MMT-005

MASTER OF SCIENCE (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCES) [MSC (MACS)]

Term-End Examination

December, 2019

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Note : *Question No. 1 is **compulsory**. Attempt any **three** questions out of Q. nos. **2** to **5**. Use of calculator is not allowed.*

1. State giving reasons whether the following statements are true *or* false : (5×2=10)

   (a) The function $f(z) = e^{-z^{-4}}$ has pole at $z = 0$.

   (b) Radius of convergence of $\sum\left(\frac{6k+1}{2k+5}\right)^k (z-2i)^k$ is $\frac{1}{2}$.

   (c) $W = \sqrt{z}$ is a multi-valued function.

   (d) $\cos|z|^2$ is analytic.

   (e) There is no $z \in C$ such that $\sin z = 10$.

2. (a) Show that if f is analytic inside and on a closed contour C and f' is continuous, then $\int_C f(z)\,dz = 0$. 3

(b) If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u + v = (x + y)(2 - 4xy + x^2 + y^2)$, then find the analytic function $f(z)$. 2

3. (a) Construct a linear fractional transformation that maps the points $-1, 1, \infty$ on the line $y = x - 1$ onto the points $1, i, -1$ on the unit circle $|w| = 1$. 3

(b) Find the Laurent series representation of the function $f(z) = \dfrac{z}{z^2 - 9}$ in the region $1 < |z + 3| < 3$. 2

4. (a) Evaluate the integral $\int_C \dfrac{dz}{z(2z - 1)(z + 2)}$, where C is the unit circle. 3

(b) Find the residue of $f(z) = \dfrac{\sin z}{z^{2n+1}}$ at $z = 0$. 2

5. Evaluate $\int_0^\infty \dfrac{dx}{1 + x^4}$. 5