M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

00875 **December, 2019**

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no. 5 is **compulsory**. Answer any **three** questions from Questions no. 1 to 4. Use of calculators is **not** allowed.*

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1. (a) Let $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - y \\ 3x - 2y \end{bmatrix}$ be a linear operator on $\mathbf{R}^2$. Find the matrix of T with respect to the basis $\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$. *2*

(b) Find the best least squares fit by a linear function to the data *3*

x	− 1	0	1	2
y	0	1	3	9

2\. (a) Check whether $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalizable or not. If it is, find an invertible matrix P so that $P^{-1}AP$ is a diagonal matrix. If A is not diagonalizable, find the Jordan form of A. *3*

(b) Find all $b \in \mathbf{R}$ for which the following matrix is positive definite : *2*

$$\begin{bmatrix} 2 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 2 \end{bmatrix}$$

3\. Find the singular value decomposition of

$$\begin{bmatrix} 2 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}.$$ *5*

4\. (a) Write all possible Jordan canonical forms for a 4×4 matrix whose only distinct eigenvalues are 1 and 2, the geometric multiplicity of 1 is two and the minimal polynomial is of degree 3. *2*

(b) Find a unitary matrix U so that $U * AU$ is a diagonal matrix, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. *3*

5. Which of the following statements are *True* and which are *False* ? Give reasons for your answers. Marks will only be given for valid reasons. *5×2=10*

(a) If T is a linear operator on a finite dimensional vector space with eigenvalue 0, then T is not onto.

(b) If A and B are $n \times n$ diagonalizable matrices, then A + B is diagonalizable.

(c) A non-zero square matrix has non-zero eigenvalues.

(d) Any two different columns of a unitary matrix are linearly independent.

(e) If the trace of a square matrix A is positive, then A is normal.